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Simple ways to evaluate the effectiveness of stimulation treatments in the layers of shales

Introduction

Popular recently issue of the shale gas has inspired the authors to present the results of the certain works carried out at the Institute of Oil and Gas over the last couple of years. It is estimated, that the silty rocks – mostly shales – constitute 75 per cent of the rocks in the geological cross-section of the holes drilled for oil and gas. Shale gas is a natural gas contained in the shale. In the last decades interest in the shale gas production has significantly increased.

Since shales have a very low permeability and low porosity, industrial production of the gas is not possible without the additional stimulation treatments, therefore they are classified as unconventional gas deposits, such as coal gas or hydrates. Natural gas can be found in the shales commonly, but the possibility of the cost-effective exploitation without stimulation procedures applied on a large scale, is very low. On a small scale, gas from shales, that are covered by a dense network of cracks, has been exploited for years, but current opportunities to exploit it are associated with the development of the new technologies, with the possibility of making horizontal long-range (up to 3000 m) wells and the hydraulic fracturing. The aim of both, fracturing and horizontal well, is to obtain the largest possible surface contact with the shale horizon. Currently in the USA around 6% of natural gas is extracted from the shale.

Shales, which contain the industrial quantities of gas, are characterized by high organic matter content (0.25÷25%) and higher natural background of gamma radiation, which is often associated with a higher content of organic carbon. The mechanism of gas storage in the shales is also complicated – most of the gas is stored in a natural network of fissures and pores, and the mechanism of gas flow is the same as in conventional porous deposits. Some part of the gas is absorbed in the form of layers of the gas molecules on the surface of the organic matter and is released as the reservoir pressure decreases. Such mechanism of gas storage is the same as in case of coal gas and it is more accurately described in the case study [6].

Besides the increased content of the organic matter, shales must have a suitable mechanical parameters to be able to perform fracturing treatment and effectively support the fracture. In Poland the shales are commonly found in the foothills of the Carpathians and in the Polish Lowland. They are often packets thick for thousands meters. While drilling in shales there are frequently observed symptoms of a gas in drilling mud, which is the evidence of the presence of gas.

Methods for assessing the effects of stimulation treatments in the shale

As it was said, profitable exploitation of the shale gas is attainable only after carrying out the complex stimulation treatments aiming at the assurance of the hydrodynamic contact with the bed on as large reservoir volume as possible. One of the most prominent stimulation process is fracturing, which is executed on a large scale in order to maximize the surface of the supported fracture. Due to the permeability of the filing material, the pressure in the supported fracture is equal to the pressure in the well, therefore the exerted depression spreads over a significant volume of the bed, allowing the flow of the gas into the fracture and the well in the amount, which justifies the cost-effective exploitation.

Since the shales constitutes the majority of rocks in the lithological cross-section of the well, thus in some geological areas the fracturing treatment will be executed at diversified depths, which may be associated with the creation of horizontal as well as vertical fractures. It is generally accepted, that at depths greater than 1000 m, only vertical fractures are created, whilst at smaller depth horizontal fractures dominate. Also the different is the geometry of the flow current lines during the flow of the gas. The following will be pointed out in the thesis: the assumptions and the utility models intended for the interpretation of the hydrodynamic tests in the wells with the vertical fracture (A), horizontal fracture (B) and in a horizontal well (C), as well as the relationships will be defined, which enable to determine the effects of the treatment by providing the proportion of the gas exploitation in the pseudo-steady state before and after the treatment for the same drawdown pressure.

Wells with a vertical fracture

Research works [4] and [3] focus on the case of vertical fracture with a length 2*a* made in a homogeneous deposit with constant thickness *h*, the permeability *k* and porosity *ϕ*. Fracture was positioned symmetrically with respect to the axis of the well, and its height was equal to the thickness of the deposit (Fig. 1).

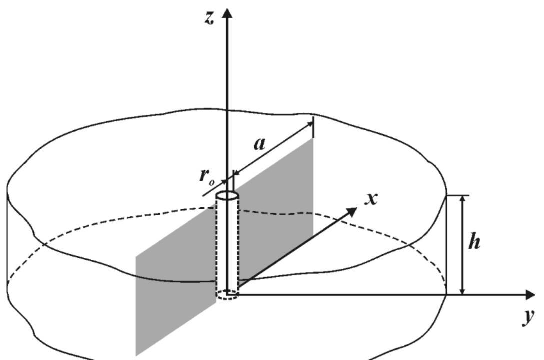

Fig. 1. The borehole with a vertical fracture

Each point of vertical fracture with dimensions 2*a* and *h* was treated as a set of sources of constant flow density *q*. Flow rate from the entire fracture Q_s, which is the sum of the flow rates of all sources, is equal to $Q_s = 2ahq$. It was assumed that the pressure *p* – like the velocity *v* of the fluid molecules are functions of the *y* and *x* coordinates. The equation describing the pressure *p* changes in the porous media during the gas flow, has the form [2].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m(p)}{\partial r}\right) = \frac{\phi\mu c}{k}\frac{\partial m(p)}{\partial t} \qquad (1)$$

where the pressure *p* was replaced by pseudo-pressure *m*(*p*) defined by the formula:

$$m(p) = 2\int_{p_b}^{p}\frac{pdp}{\mu Z} \qquad (2)$$

The following initial and boundary conditions were assumed:

$$p = p_o \quad \text{for} \quad t = 0 \text{ and for any point } P(x, y) \qquad (3)$$

and:

$$\lim_{r\to\infty} p = p_o \qquad \text{for any time } t \geq 0 \qquad (4)$$

In [3] it was proved, that in case of gas flow to the well with a vertical fracture with a constant flow rate Q_N for a short times of flow for *t* responding to the condition:

$$t[\min] \leq 1.0416\cdot 10^4\,\frac{\phi\mu c a^2}{k} \qquad (5)$$

the equation describing the changes of the gas pressure as a function of time is:

$$m(p_o) - m(p_{dr}(t)) = 0.799\,\frac{Q_N T}{ah}\sqrt{\frac{1}{\phi k(\mu c)_p}}\times$$
$$\times\left(\sqrt{t} + 230.3a\sqrt{\frac{\phi(\mu c)_p}{k}}S\right) \qquad (6)$$

while the equation for pressure build-up after the closure of the well is:

$$m(p_o) - m(p_{odb}(t)) = 0.799\,\frac{Q_N T}{ah}\sqrt{\frac{1}{\phi k(\mu c)_p}}\times$$
$$\times\left(\sqrt{t_z + \Delta t} - \sqrt{\Delta t}\right) \qquad (7)$$

In case of long fracture and very low permeability, which is a case in shales, formulas (5) and (6) may be valid for a long time. For example, assuming a porosity $\phi = 0.01$, the product of viscosity and compressibility $(\mu c)_p = 5{,}3\cdot 10^{-5}$ cP/MP, $a = 50$ m, $k = 0.01$ mD, $h = 15$ m in a formula (5) indicates, that the relations (6) and (7) will be valid for $t \leq 1380$ minutes, which is – 23 hours. Formulas (6) and (7) are used for interpretation of the well tests data for wells with a vertical fracture, that is, to determine the permeability of the deposit (shales), when the length of the fracture *a* is known or they allow us to calculate length of fracture a if permeability is known from other sources. There is a general tendency in the industry to give up the use of theoretically correct pseudo-pressures in favour of pressures – in that case relations (6) and (7) are:

$$p_o^2 - p_{dr}^2(t) = 0.0253\frac{Q_N \mu ZT}{ah}\sqrt{\frac{1}{\phi k(\mu c)_p}} \times \left(\sqrt{t} + 72.85a\sqrt{\frac{\phi(\mu c)_p}{k}}S\right) \quad (8)$$

and:

$$p_o^2 - p_{odb}^2(t) = 0.0253\frac{Q_N \mu ZT}{ah}\sqrt{\frac{1}{\phi k(\mu c)_p}} \times \left(\sqrt{t_z + \Delta t} - \sqrt{\Delta t}\right) \quad (9)$$

In formulas (8) and (9) the pressure is expressed in MPa.

It was proved in [4] and [3] that for the pseudo-steady state of the gas flow to the well the following relation is valid:

$$\frac{Q_s}{Q_o} = \frac{\ln\frac{r_e}{r_o\sqrt{e}}(p_o - p_{dr})}{\ln\frac{r_e}{\frac{a}{e}\sqrt{e}}(p_o - p_{dr})} \quad (10)$$

where:
Q_s – steady-state gas flow rate to vertically fractured well,
Q_o – steady-state gas flow rate to vertical well,
r_o – drainage radius,
p_o – reservoir pressure,
p_{dr} – bottomhole flowing pressure,
e – base of the natural logarithm,
a – fracture length.

Equation (10) indicates that for the same drawn-down pressure in vertically fractured and standard well the creation of vertical fracture of length 2a is approximately equivalent to enlargement of the well radius to the value (a/e). There is also known Prats's formula, according to which:

$$\frac{Q_s}{Q_o} = \frac{\ln\frac{r_e}{r_o}(p_o - p_{dr})}{\ln\frac{r_e}{a}(p_o - p_{dr})} \quad (11)$$

Example

From the bed of shales with thickness h = 80 m, the permeability of which has been already defined on the basis of the laboratory research as equal to k = 0.01 mD, the gas had been exploited with flow rate 10 Nm^3/min. Because of low flow rate the hydraulic fracturing was performed, as a result of which the vertical fracture was created as in Fig. 1. The length of the fracture a is unknown. Other data are as follows: porosity ϕ = 0.01, $(\mu c)_p$ = 5,3 · 10^{-4} cP/MPa, p_o = 15 MPa, Z = 0.9, T = 313 K, μ = 0.018 cP, re = 400 m. After the treatment the gas production was resumed Q_N = 15 m^3/min and changes of draw-down pressure vs.t were recorded. The measurement results are summarized in the table below.

Table 1

p_{dr} [MPa]	t [min]	p_{dr} [MPa]	t [min]
14.78	10	13.61	360
14.68	20	13.49	420
14.45	60	13.39	480
14.22	120	13.19	600
14.04	180	12.99	720
13.87	240	12.82	840
13.74	300	12.72	920

The equation (8) indicates that the points $p_o^2 - p_{dr}^2(t)$ vs. $\sqrt{t}$ should plot along a straight line with slope:

$$m = 0.0253\frac{Q_N[\text{Nm}^3/\min]\mu[\text{cP}]ZT[K]}{a[m]h[m]} \times \sqrt{\frac{1}{\phi k[\text{mD}](\mu c)_p[\text{cP/MPa}]}} \quad (12)$$

The graph depicting relations $p_o^2 - p_{dr}^2(t)$ vs. $\sqrt{t}$ (Fig. 2) indicates that the slope m of the straight line is equal to 2.089 MPa^2/min$^{1/2}$, and thus we obtain from (12):

$$a = 0.0253\frac{Q_N[\text{Nm}^3/\min]\mu[\text{cP}]ZT[\text{K}]}{m[\text{MPa}^2/\min^{1/2}]h[\text{m}]} \times \sqrt{\frac{1}{\phi k[\text{mD}](\mu c)_p[\text{cP/MPa}]}} = 0.0253\frac{(25)(0.018)(0.9)(313)}{(2.089)(80)} \times \sqrt{\frac{1}{(0.01)(0.01)(5.3)(10^{-4})}} \cong 50\ \text{m}$$

and therefore the length of a fracture is equal to about 50 m.

Knowing a, is it necessary to check, whether the use of formula (8) to identify a was justified, that is whether the period of its validity has not passed, since it is correct for t responding to the condition (5):

Table 2

$p_o^2 - p_{dr}^2(t)$ [MPa^2]	$\sqrt{t}$ [min$^{1/2}$]	$p_o^2 - p_{dr}^2(t)$ [MPa^2]	$\sqrt{t}$ [min$^{1/2}$]
6.55	3.16	39.76	18.97
9.50	4.47	43.02	20.49
16.20	7.75	45.70	21.91
22.80	10.95	51.02	24.49
28.02	13.42	56.26	26.83
32.60	15.49	60.65	28.98
36.20	17.32	63.20	30.33

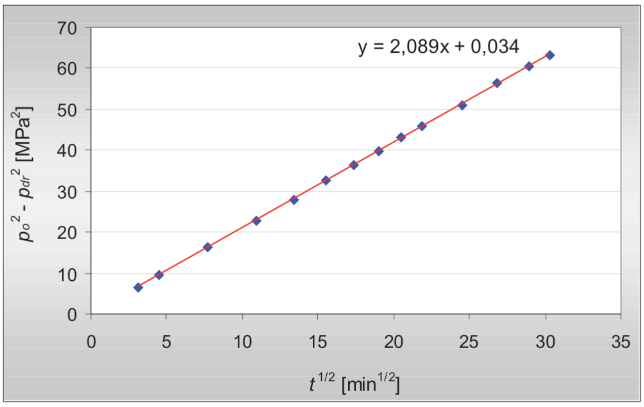

Fig. 2. The relation of $p_o^2 - p_{dr}^2$(t) vs. $\sqrt{t}$

$$t[\text{min}] \le 1.0416 \cdot 10^4 \frac{\phi\mu c a^2}{k} \le 1.0416 \cdot 10^4 \times$$

$$\times \frac{0.01 \cdot 5.3 \cdot 10^{-5} \cdot 50^2}{0.01} \le 1380 \text{ minutes, that is 23 hours}$$

Thus we can see that use of equation (8) was justified. For the pseudo-steady state of flow the relation (10) is valid. It indicates that the ratio of gas flow from vertically fractured well to gas flow from standard vertical well for the same bottomhole pressure is defined by the following equation:

$$\frac{Q_s}{Q_o} = \frac{\ln \dfrac{400}{0.108\sqrt{2.7183}}}{\ln \dfrac{400}{\dfrac{50}{2.7813}\sqrt{2.7183}}} \cong 3$$

The wells with a horizontal fracture

The state of stress in shallow depths favour creation of horizontal fractures. The existence of a horizontal fractures changes the geometry of the flow current lines around the well and the nature of relations of the bottom-hole flowing pressure vs. time. All these issues were examined in research works [7] and [8].

The following case was taken into consideration: In a vertical well drilled through the homogeneous porous layer with thickness $2h$, the permeability k and porosity ϕ, at the mid point of pay horizon the circle shaped horizontal fracture was made (with an unknown radius R), centre of which lies in the axis of the well (Fig. 3).

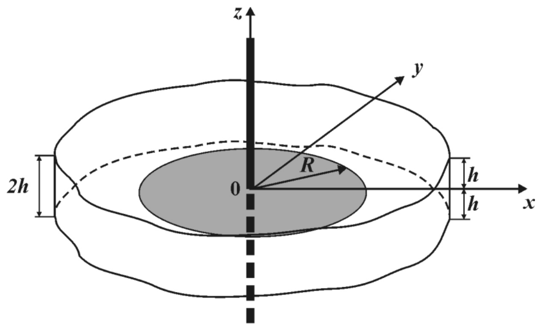

Fig. 3. Illustrative diagram of the horizontal fracture

It was assumed that:

- initial reservoir pressure p_0 prevails throughout the reservoir,
- at the time $t = 0$ production is started with constant flow rate Q,
- at large distance from the well there is the reservoir pressure p_o for each time t.

Each point of horizontal fracture was treated as a point source, which is acting for $t > 0$; moreover it was assumed, that in the case of spherical flow to a source located in the point $P(\xi, \eta, \mu)$ the pressure satisfies the equation:

$$\frac{\phi \cdot \mu \cdot c}{k}\frac{\partial p}{\partial t} = \nabla^2 p + \frac{\mu}{k} q(t')\partial(x-\xi)\partial(y-\eta)\partial(z-\mu) \qquad (13)$$

where the point source is modelled using Dirac's distribution. In the notation of equation (13) the Darcy's law was included:

$$\lim_{r \to 0} r^2 \left(\frac{\partial p}{\partial r}\right) = \frac{q\mu}{4\pi k} \qquad (14)$$

where:

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\mu)^2} \qquad (15)$$

In [7] and [8] it was proved that for:

$$t \le \frac{R^2 \phi\mu c}{16k} \qquad (16)$$

the relation combining the bottom-hole flowing pressure versus time of flow during production from the well with a horizontal fracture with a constant gas flow rate Q_N looks like:

$$p_0^{2}[\mathrm{MPa}^2]-p_{dr}^{2}[\mathrm{MPa}^2]= \\ =0.0161\frac{Q_N[\mathrm{Nm}^3/\min]\mu[\mathrm{cP}]ZT[\mathrm{K}]}{R^2[\mathrm{m}^2]}\times \\ \times\sqrt{\frac{t[\min]}{\mu[\mathrm{cP}]\phi k[\mathrm{mD}]c[1/\mathrm{MPa}]}} \tag{17}$$

It was demonstrated [7] that in the infinitely long period of time, in the case of medium with unlimited extent in all directions (which in practice corresponds to deposits with large thickness and long times of flow) the following relation holds:

$$p_0^{2}[\mathrm{MPa}^2]-p_{dr}^{2}[\mathrm{MPa}^2]=1.843\times \\ \times\frac{Q_N[\mathrm{Nm}^3/\min]\mu[\mathrm{cP}]ZT[\mathrm{K}]}{R[\mathrm{m}]k[\mathrm{mD}]} \tag{18}$$

Knowing all quantities appearing in (18) we can calculate the gas flow rate in the well with a horizontal fracture for the assumed flowing pressure. For pseudo-steady state of flow the flowing relation holds:

$$\frac{Q_s}{Q_o}=\frac{R}{h}\ln\frac{r_e}{r_o\sqrt{e}} \tag{19}$$

Example (hypothetical)

In shale horizon with thickness h = 80 m, the permeability of which was laboratory-defined as equal to 0.02 mD, the horizontal fracture was created (shallow depth) and gas production was started with the flow rate 20 Nm^3/min. The bottomhole flowing pressure versus time was recorded. These data are summarized in the table below. Rest of data are as follows: initial reservoir pressure p_o = 15 MPa, gas viscosity μ = 0.018 cP, compressibility factor Z = 0.9, reservoir temperature T_z = 313 K, porosity ϕ = 0.01, compressibility factor of the system: rock and fluids saturating it c = 0,0294 1/MPa, drainage radius r_e = 200 m, well radius r_o = 0.108 m.

Table 3

p_{dr} [MPa]	t [min]	p_{dr} [MPa]	t [min]
14.82	5	13.79	300
14.74	15	13.67	360
14.66	25	13.56	420
14.47	60	13.45	480
14.25	120	13.26	600
14.07	180	13.05	720
13.92	240		

Relation of $p_o^2-p_{dr}^2(t)$ vs. $\sqrt{t}$ is as follows:

Table 4

$p_o^2-p_{dr}^2(t)$ [MPa^2]	$\sqrt{t}$ [min$^{1/2}$]	$p_o^2-p_{dr}^2(t)$ [MPa^2]	$\sqrt{t}$ [min$^{1/2}$]
5.73	2.36	34.84	17.32
7.73	3.87	38.13	18.97
10.08	5.00	41.13	20.49
15.62	7.75	44.10	21.91
21.94	10.95	49.17	24.49
27.04	13.42	53.70	26.83
31.23	15.49		

Slope of the graph $p_o^2-p_{dr}^2(t)$ vs. $\sqrt{t}$ (fig. 4.) is equal to $m = 2.008$. Equation (17) indicates that the slope is equal to:

$$m=2.008=0.0161\frac{Q\mu ZT}{R^2}\sqrt{\frac{1}{\phi k\mu c}}$$

and thus the radius R of the fracture is:

$$R=\sqrt{\frac{0.0161}{2.008}}\sqrt{\frac{Q[\mathrm{Nm}^3/\min]\mu[\mathrm{cP}]ZT[\mathrm{K}]}{\sqrt{\phi\mu[\mathrm{cP}]k[\mathrm{mD}]c[1/\mathrm{MPa}]}}}= \\ =0.0895\sqrt{\frac{20\cdot0.018\cdot0.9\cdot313}{\sqrt{0.01\cdot0.018\cdot0.02\cdot0.0294}}}\cong 50\ \mathrm{m}$$

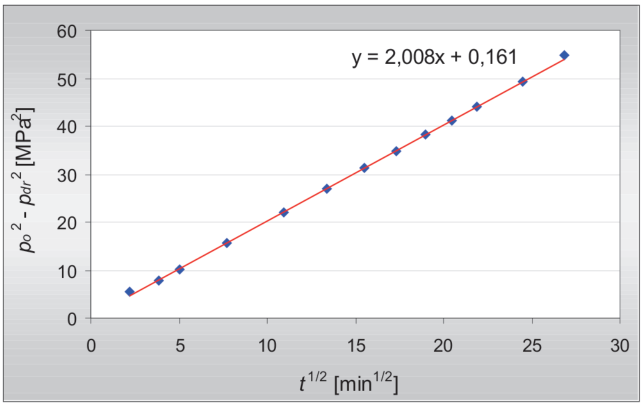

Fig. 4. The chart of relations $p_o^2-p_{dr}^2(t)$ to $\sqrt{t}$

Then, we have to verify, whether it was justified to use the equation (17), which is correct for t meeting the condition (16), having the following form in industrial units:

$$t[\min]\le 1.041\cdot10^3\,\frac{R^2[\mathrm{m}^2]\phi\mu[\mathrm{cP}]c[1/\mathrm{MPa}]}{k[\mathrm{mD}]}$$

that is:

$$t[\min]\le 1.041\cdot10^3\,\frac{50^2\cdot0.01\cdot0.018\cdot0.0294}{0.02}\le 689\ \min$$

As shown the use of (17) was justified.

After infinitely long time of flow, assuming the unlimited extent in all directions, which is in practice after a long period of production of very thick horizon, we can determine flow rate from well with horizontal fracture [7]. Assuming given data and the bottom-hole flowing pressure, for instance, p_{dr} = 8 MPa, we receive:

$$Q = \frac{\left(p_0^{\ 2} - p_{dr}^{\ 2}\right)[\text{MPa}^2]R[\text{m}]k[\text{mD}]}{1.843ZT[\text{K}]\mu[\text{cP}]} = \frac{(225-64)\cdot 50\cdot 0.02}{1.843\cdot 0.9\cdot 313\cdot 0.018} \approx 17.2\ \text{Nm}^3/\text{min}$$

Obtained results should be treated with caution, because the reservoir pressure changes with time. Furthermore the above relation was derived assuming the unlimited extent of reservoir in all directions. The formula (19) shows that for the pseudo-steady state of flow, the flow rate caused by the horizontal fracture will be equal to:

$$\frac{Q_s}{Q_o} = \frac{R}{h}\ln\frac{r_e}{r_o\sqrt{e}} = \frac{50}{80}\ln\frac{200}{0.108\sqrt{2.781}} \cong 4.4$$

The horizontal well (C)

Drilling the horizontal wells is aimed to get the best possible hydrodynamic contact with the shale horizon. The issue of the quantitative description of the gas flow to the horizontal well is discussed in [5], in which it was assumed that:

- the horizontal well with r_o radius was drilled in horizontal shale horizon with thickness h; the well was completed by perforation along the distance a; the origin of XY coordinates is situated at the beginning of completed segment and OX line runs along the well axis (see Figure 5);

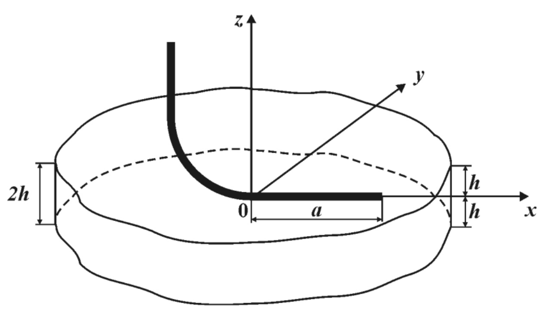

Fig. 5. The horizontal borehole in a productive layer

- at the initial time the bed has a initial reservoir pressure p_o,
- at the time $t = 0$ the well begins produce gas with flow rate $Q(t)$,
- at the large distance from the well there is initial reservoir pressure for each t,
- we consider the transient flow to the horizontal segment of the well with the length a,
- each of the points of the horizontal segment of the well is treated as a point source operating for $t > 0$.

In [7] it was proved that in case of the spherical flow to the source situated in the point $P(\xi, \eta, \mu)$ in the unlimited region, the pressure satisfies the equation:

$$\frac{\phi\cdot\mu\cdot c}{k}\frac{\partial p}{\partial t} = \nabla^2 p + \frac{\mu}{k}q(t)\partial(x-\xi)\partial(y-\eta)\partial(z-\mu) \quad (20)$$

where the point source is modeled using Dirac's distribution. In the notation of equation (20) the Darcy's law is included:

$$\lim_{r\to 0} r^2\left(\frac{\partial p}{\partial r}\right) = \frac{q\mu}{4\pi k} \quad (21)$$

where:

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\mu)^2} \quad (22)$$

In [5] there were derived relations between the bottom-hole flowing pressure versus time of flow which have rather complicated character. This study also proved that for:

$$7.23\cdot 10^{-1}\frac{\phi\mu[\text{cP}]c[1/\text{MPa}]a^2[\text{m}^2]}{k[\text{mD}]} \geq t[\text{days}] \geq$$
$$\geq 2.8938\cdot 10^{2}\frac{\phi\mu[\text{cP}]c[1/\text{MPa}](r_o^{\ 2}+h^2)[\text{m}^2]}{k[\text{mD}]} \quad (23)$$

the equation combining the bottom-hole flowing pressure and time in the case of gas flow with constant flow rate Q to the horizontal well looks like shown below:

$$\left(p_o - p_{dr}\right)_s = \frac{Q_s\mu}{4\pi ak}\ln\frac{4kt}{\gamma\phi\mu c r_o h} \quad (24)$$

where γ = 1.781 – is Euler's constant. The set specified by inequality (23) is not an empty set for $a/h > 20$.

Comparing (24) to the equation, which describe changes of the bottom-hole flowing pressure caused by influx of the reservoir fluid to the vertical well, the following formulae is obtained:

$$\left(p_o - p_{dr}\right)_p = \frac{Q\mu}{4\pi hk}\ln\frac{4kt}{\gamma\phi\mu c r_o^{\ 2}} \quad (25)$$

It can be easily proved, that by assuming the same drawn-down pressure the following is valid:

$$\frac{Q_s}{Q}=\frac{a}{h\left(1-\dfrac{\ln\dfrac{h}{r_o}}{\ln\dfrac{k[\text{mD}]t[\text{days}]}{\phi\mu[\text{cP}]c[1/\text{MPa}]r_o^2[\text{m}^2]}-1.639}\right)} \tag{26}$$

where Q_s and Q are correspondingly flow rates of the horizontal and vertical wells for the same draw-down pressure.

If the permeability of the shales is not known, it could be determined on the basis of the rate of the bottom-hole flowing pressure decrease. Initial equation is the relation derived in [5]:

$$\frac{dp_{dr}}{dt}\left[\frac{\text{MPa}}{\text{min}}\right]=-0.461\frac{Q_N[\text{Nm}^3/\text{min}]\mu[\text{cP}]ZT[\text{K}]}{p_o[\text{MPa}]a[\text{m}]k[\text{mD}]t[\text{min}]}\times$$

$$\times\left(1+e^{-4.17\cdot 10^3\frac{\phi\mu[\text{cP}]c[1/\text{MPa}]H^2[\text{m}^2]}{k[\text{mD}]t[\text{min}]}}\right) \tag{27}$$

The formula (27) holds for t meeting the condition:

$$1.0417\cdot 10^3\cdot\phi\mu[\text{cP}]c[1/\text{MPa}]a^2[\text{m}^2]>v[\text{mD}\cdot\text{min}]>$$

$$>-4.1667\cdot 10^5\cdot\phi\mu[\text{cP}]c[1/\text{MPa}]r_o^2[\text{m}^2] \tag{28}$$

Substituting (27) v instead of kt we can construct a theoretical graph of relations dp_{dr}/dt vs. v for v from the range specified by the inequality (28). If, for instance, after the time t the rate of the pressure decline is dp_{dr}/dt and corresponding magnitudes is v, so we can compute the permeability k as $v = kt$ because time t is known. This procedure is exemplified below:

Example

At some draw-down pressure the average flow rate of gas from the deposit of shales with thickness h = 120 m is 1.5 Nm^3/min in the case of the vertical well. In order to increase the gas production the shale horizon was completed with horizontal well along the length a = 800 m. Other data are as follows: porosity $\phi = 0.01$, gas viscosity μ = 0.018 cP, permeability k = 0.01 mD, the compressibility coefficient of the deposit rock and fluids saturating it c = 0.0294 1/MPa, the well radius r_o = 0.108 m

Formula (24), and therefore (26), holds for t from the range specified by the inequality (23), i.e. for:

$$245\text{ days} > t > 61\text{ days}$$

As results from (26) the Q_s/Q ratio will be equal to approximately 62 for t = 61 days and 60 for t = 245 days, so the 800 m horizontal completion will increase gas flow rate about 60 times while maintaining the same draw-down pressure as in vertical well. In the case of shorter horizontal segment the Q_s/Q ratio will be correspondingly lower.

Example of computing permeability in the horizontal well (hypothetical)

The length of horizontal section of a well is equal to a = 800 m. The gas flow rate was Q_N = 90 Nm^3/min. The reservoir pressure was equal to p_o = 15 MPa. Other data are identical as in the previous example.

The procedure is as follows:

- Using the inequality (28) we determine the range of v, for which the formula (27) is valid. We receive from (28) after substituting the adequate data:

$$3528 > v > 0.026$$

- For magnitude v from the above range we build a theoretical graph dp_{dr}/dt vs. v, shown for a given set of data in the figure 6 below (for each set of details, the graph will be different).
- Measurements taken during the production period are shown together with values read from the graph v and calculated permeabilities k.

Permeability can be also calculated with formula (24) that is: converting it for the flow of gas and recording relation $p_o^2 - p_{dr}^2$ versus log t, similarly to the interpretation of standard well tests. Above mentioned method enables however the calculation of k in much broader time interval.

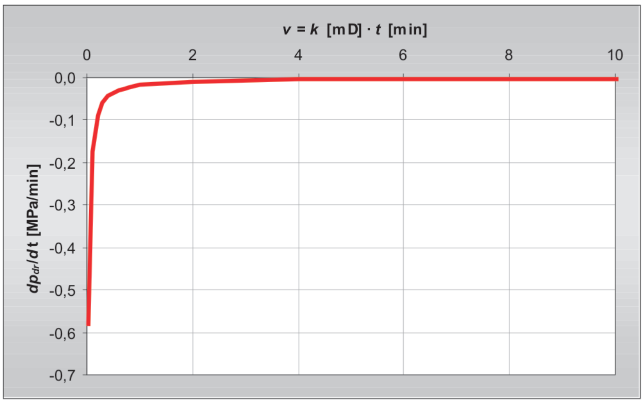

Fig. 6. The graph of relations dp_{dr}/dt vs. v for the data from the example

Table 5

t [min]	dp_{dr}/dt [MPa/min]	v [mD·min]	k [mD]
50	0,0351	0,5	0,01
250	0,0072	2,5	0,01
500	0,0041	5,0	0,01
1000	0,0025	10,0	0,01

Conclusions

The relations described above can provide the assessment of the increase of production in low permeability reservoirs caused by creation of vertical and horizontal fractures, as well as they allow to compare gas flow rates in cases of making the reservoir available by horizontal and vertical well. Formulas for Q_s/Q are approximate, but presumably sufficient for engineering purposes, providing they meet the assumptions accepted in particular model.

One should remember, that in the deposit of shales completed using the horizontal well, the several fractures are created and productivity increase caused by the realization of such a complex operations is not a simple sum of the productivity gained due to a single treatment, because trajectories of flow lines are different for each case. Moreover, the gas deposits in the shales are frequently completed by several horizontal wells drilled from a single vertical well. In above-mentioned cases, assessment of the completion results would require the construction of much more complicated numerical models.

As shown above, the fracturing treatment could cause the multiple increase of production when compared with the standard vertical well. However, we should keep in mind that such a big increase of production has to be referred to a very low initial level of a few Nm^3/min due to the extremely low permeability of the shales.

Symbols

Q_s – flow rate after the fracturing
Q_o – flow rate of the vertical well
a – length of the fracture or a length of the horizontal segment of the well
h – thickness of the reservoir
k – permeability
μ – viscosity
T – temperature
Z – factor taking into account real gas behavior
c – coefficient of compressibility
r_o – radius of the well
p_o – the initial reservoir pressure
p_{dr} – bottom-hole flowing pressure
r_e – drainage radius
R – radius of the horizontal fracture
t – time

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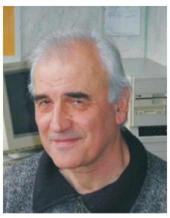
Literature

[1] Kącki E.: *Termokinetyka*. WNT, Warszawa, 1967.
[2] Sneddon I.: *Równania różniczkowe i cząstkowe*. PWN, Warszawa, 1981.
[3] Szpunar T., Budak P., Herman Z.: *Metodyka badań hydrodynamicznych w otworach wiertniczych wykonywanych dla pozyskania gazu ze złóż węgla kamiennego*. Dok. INiG, Kraków, 1992.
[4] Szpunar T., Budak P.: *Interpretacja danych ciśnieniowych początkowego okresu przypływu płynu do otworu przechodzącego przez warstwę produktywną ze szczeliną poziomą*. Nafta-Gaz, nr 9, 2009.
[5] Szpunar T.: *Interpretacja krzywych przypływu i odbudowy w odwiertach ze szczeliną poziomą*. Nafta-Gaz, nr 1, 1993.
[6] Szpunar T.: *Interpretacja wyników badań hydrodynamicznych w odwiertach poziomych*. Nafta-Gaz, nr 9–10, 1992.
[7] Szpunar T.: *Metoda interpretacji krzywych spadku i odbudowy ciśnienia w odwiertach ze szczeliną pionową*. Górnictwo, Zeszyt 2, 1987.
[8] Szpunar T.: *Wpływ szczeliny pionowej na zmiany ciśnienia w otworze przy nieustalonym przepływie cieczy słabościśliwej*. Górnictwo, Zeszyt 4, 1986.



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