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A modified concept for carrying out and interpretation of multi-rate gas well deliverability testing, using flow rate control choke

This paper presents a modified procedure for the execution and interpretation of results for multi-rate gas well deliverability testing. The proposed procedure requires using a flow regulation device, which allows for the control of the gas flow rate. The exponent in conventional gas well deliverability equation is allowed to be different from 2. Provided are example interpretations of the multi-rate gas wells deliverability tests carried out according to the proposed procedure using the real world data.

Key words: multi-rate production test, laminar/turbulent flow coefficients, deliverability, absolute open flow potential.

Zmodyfikowany sposób przeprowadzania i interpretacji wyników wielocyklowego testu produkcyjnego odwiertu gazowego

Przedstawiono zmodyfikowaną metodykę realizacji i interpretacji wyników wielocyklowego testu przypływu do odwiertu gazowego. Do przeprowadzenia testowań odwiertu gazowego zgodnie z proponowaną metodyką wymagane jest użycie zwężki o regulowanym przelocie umożliwiającej uzyskanie żądanego natężenia przepływu gazu. Dopuszczono różną od 2 wielkość wykładnika potęgowego w tzw. formule dwuczłonowej. Załączono przykłady obliczeniowe dla odwiertów gazowych.

Słowa kluczowe: test produkcyjny wielocyklowy, współczynniki formuły dwuczłonowej, potencjalne natężenie wypływu gazu.

Introduction

The conventional multi-rate gas well test consists of the production of the well at various stabilized flow rates and measuring the stabilized sand face pressure at the end of each flow cycle. After each flow cycle is completed, the well is closed for pressure stabilization until pressure builds up to its original value which is equal to the average pressure within the drainage area of the well being tested. The aim of gas well testing is to measure the production capabilities at the specific conditions of a reservoir and bottom hole flowing pressure. The multi-rate gas well test enables calculation of the maximum flow potential of the well (Q_{abs}) and generates the inflow performance curve, which indicates the relation between surface flow rate and bottom hole flowing pressure for a given value of average pressure, within the drainage area of the well.

Besides conventional multi-rate gas well tests for which build up periods are continued until stabilization of pressure is reached, there are several other deliverability testing methods developed to shorten the testing time including:

- Flow after flow tests, which consist of flowing the well at a series of constant flow rates and measuring the stabilized sand face pressure; the flowing periods are not followed by pressure build up periods so the final flow pressure of the preceding cycle is the initial pressure of the next flow cycle.
- An isochronal test, which shortens testing time by skipping pressure stabilization for flow cycles, while build up pressure periods are continued until the pressure is stabilized, to the average pressure within the drainage area.

- A modified isochronal test in which the flow periods are of equal duration. Likewise, the pressure build up periods are also of equal duration, but not necessarily the same as the flow periods.

The equation which relates gas flow rate, average pressure within drainage area and stabilized sand face flowing pressure developed by Houpeurt is commonly used for interpretation of conventional multi-rate gas well deliverability tests and its above mentioned versions. The Houpeurt equation is the theoretical one contrary to Rawlins-Shellhard formula which is an empirical equation. The Houpeurt theoretical equation which is generally used for high flow rate wells has the following form:

$$p_s^2 - {p_{bhfp}}^2 = aQ + bQ^2 \tag{1}$$

where:

p_s – average pressure within drainage area of the well or initial flowing pressure,

p_{bhfp} – stabilized bottom hole flowing pressure (stabilized sand face flowing pressure),

Q – stabilized flow rate,

a, b – coefficients.

If a and b are known the absolute open flow potential Q_{abs} can be evaluated and the inflow performance curve can be generated, enabling calculation of the sand face drawn down pressure needed to produce the required gas flowing rate. The problems related to execution and interpretation of the conventional gas well deliverability tests, are widely discussed in literature and well known to petroleum engineers and thus there is no reason to discuss them here.

Modified concept for carrying out and interpretation of multi-rate gas well test

In practice Eq. 1 doesn't always precisely describe the relation between gas flow rate Q, stabilized sand face pressure, and average pressure within the drainage area of the well being tested. This is mostly caused by a phenomenon occurring within the well bore zone such as:

- precipitation of gas condensates due to pressure and temperature drop,
- difference between permeability of the well bore zone and reservoir,
- installation of the sand control screens,
- well completion (perforation or open hole completion).

One should also recall that several simplifying assumptions were made in derivation of Eq. 1 which are not satisfied in a real world scenario, which causes that sometimes the data of one or more flowing cycles must be rejected if they drift away from the linear trend of $(p_s^2 - {p_{bhfp}}^2)/Q$ vs. Q or when the correlation of data is poor. If this happens the authors propose to modify the segment describing the impact of flow turbulence and use a slightly different form of Eq. 1, hoping that it would improve the flexibility and accuracy of the interpretation.

$$p_s^2 - {p_{bhfp}}^2 = aQ + bQ^n \tag{2}$$

There are three unknowns a, b and n because n is allowed to be different than 2 if measurements indicate so. The calculation of Q_{abs} and construction of the IPR curve using standard procedure used in case of Eq. 1 (i.e. finding a, b and n using the least squares method) is extremely inconvenient mathematically. If Eq. 2 is to be used the following procedure is recommended for the conduction and interpretation of multi-rate gas well deliverability tests. Let's assume that the flow rates are consecutively growing for each succeeding flow cycle.

1. Start flowing the well with the first flow rate Q_1 and record the corresponding stabilized sand face flowing pressure p_{bhfp1}.
2. Shut in the well for pressure stabilization to original average pressure within drainage area p_s.
3. Repeat procedure indicated in steps 1) and 2) for Q_N where Q_N is some maximum flow rate planned.
4. Calculate the flow rates of intermediate flow cycles using the following formula.

$$Q_{N-i} = Q_1^{\frac{i}{N-1}} Q_N^{\left(1-\frac{i}{N-1}\right)} \tag{3}$$

where $i = 1, 2, \ldots, N-2$; N – number of flow cycles.

5. Carry out the series of flow cycles with intermediate flow rates and record the stabilized sand face flowing pressures corresponding to each flow rate. Each flow cycle should be followed by a pressure build up period to original average reservoir pressure p_s.
6. Calculate C_i for all flow rates:

$$C_1 - (p_s^2 - {p_{bhfpi}}^2)/Q_i \tag{4}$$

where $i = 1, 2, \ldots, N$.

7. Calculate a using the following formula:

$$a = \frac{1}{N-2}\sum_{i=1}^{N-2}\left(\frac{C_i C_{i+2} - C_{i+1}^2}{C_i + C_{i+2} - 2C_{i+1}}\right) \tag{5}$$

where $i = 1, 2, \ldots, N-2$.

A) Present the Eq. 1 in a following form:

$$\log(C_i - a) = (n-1)\log Q_i + \log b \tag{6}$$

Mark the $\log(C_i - a)$ vs. $\log Q_i$ on rectangular coordinates. The values of $\log(C_i - a)$ vs. $\log Q_i$ should plot along the straight line – with slope (n – 1) enabling calculation of n – which intersect the ordinate axis in $m = \log b$ for $Q = 1$ enabling calculation of b.

The n and b values may be also calculated using the following formulas:

$$n = 1 + \frac{N\sum_{i=1}^{N}\log(C_i - a)\log Q_i - \sum_{i=1}^{N}\log(C_i - a)\sum_{i=1}^{N}\log Q_i}{N\sum_{i=1}^{N}(\log Q_i)^2 - \left(\sum_{i=1}^{N}\log Q_i\right)^2} \tag{7}$$

$$\log b = \frac{\sum_{i=1}^{N}\log(C_i - a)\sum_{i=1}^{N}(\log Q_i)^2 - \sum_{i=1}^{N}\log(C_i - a)\log Q_i\sum_{i=1}^{N}\log Q_i}{N\sum_{i=1}^{N}(\log Q_i)^2 - \left(\sum_{i=1}^{N}\log Q_i\right)^2} \tag{8}$$

where $i = 1, 2, \ldots, N$.

or

B) Calculate n using the following formula

$$n = 1 + \frac{1}{N-1}\sum_{i=1}^{N-1}\frac{\log\frac{C_i - a}{C_{i+1} - a}}{\log\frac{Q_i}{Q_{i+1}}} \tag{9}$$

where $i = 1, 2, \ldots, N - 1$.

Knowing n calculate b using the following formula

$$b = \frac{1}{N}\sum_{i=1}^{N}\frac{C_i - a}{Q_i^{n-1}} \tag{10}$$

The procedure shown above may also be applied when the gas pseudo pressures $m(p)$ are used instead of p^2 in Eq. 1. This will only require some simple modification in Eq. 4.

Equation (2) is the more general form of the theoretical Eq. (1). If the p_{bhfp_i} and Q_i perfectly satisfy the theoretical equation $p_s^2 - p_{bhfp_i}^2 = aQ_i + bQ_i^2$ and Q_i satisfies condition given by Eq. (3) then the a, b and n coefficients calculated using the herein presented approach and conventional method will be practically the same (i.e. n will be equal 2), which can be easily demonstrated using simple calculations.

Indeed, let us consider Example 1 given below for which, a and b coefficients calculated using the conventional method are $a = 0.5713$ MPa²/(Nm³/min) and $b = 0.0013$ MPa²/(Nm³/min)². The flow rates Q_i which satisfy Eq. 3 and corresponding theoretical values of p_{bhfp_i} (calculated using formula $p_{bhfp_i} = \sqrt{p_s^2 - 0.5713Q_i - 0.0013Q_i^2}$) are given in columns 1 and 2 respectively. The auxiliary coefficient C_i (Eq. 4) is given in column 3 of the Table 1 below.

Table 1.

N	Q_i [Nm³/min]	p_{bhfpi} [MPa]	C_i MPa²/(Nm³/min)
1	88.49	12.310	0.6865
2	108.30	11.626	0.7120
3	132.55	10.664	0.7436
4	162.23	9.240	0.7823

The coefficients a, b and n calculated using the proposed modified approach are as follows:

$$a = \frac{1}{N-2}\sum_{i=1}^{N-2}\left(\frac{C_i C_{i+2} - C_{i+1}^2}{C_i + C_{i+2} - 2C_{i+1}}\right) = 0.5713$$

$$n = 1 + \frac{1}{N-1}\sum_{i=1}^{N-1}\frac{\log\frac{C_i - a}{C_{i+1} - a}}{\log\frac{Q_i}{Q_{i=1}}} = 2$$

$$b = \frac{1}{N}\sum_{i=1}^{N}\frac{C_i - a}{Q_i^{n-1}} = 0.0013$$

i.e. they are the same as those calculated using the conventional method. The interpretation of multi-rate flow tests using the conventional method, means the adaptation of measurements "by force" to theory, while the proposed modified approach seems to be more flexible and better reflects the bottom hole pressure vs. flow rate relation.

Examples of interpretation

The proposed multi-rate gas well deliverability tests have never been used before, but among hundreds of test samples the authors succeeded in finding a dozen or so, for which flow rates in intermediate cycles were coincidentally almost equal to that indicated by Eq. 3. In the Tables below in the column entitled "gas flow rate" the flow rates recorded are followed by flow rates required by Eq. 3 provided in brackets. The fact that multiple datasets which satisfied the flow rate requirements given by Eq. 3 yielded reasonable results, support the usefulness of the present approach.

Example no 1

Well X1 (*data from Theory and practice of the testing of gas wells. Calgary 1975, Chapter 3, page 25*).
Number of flow cycles $N = 4$.
Average pressure within drainage area $p_s = 14.57$ MPa (corrected value).

Table 2.

Number of cycle	Gas flow rate Q_i [Nm³/min]	Stabilized sand face pressure p_{bhfpi} [MPa]
1	88.49	12.300
2	110.12 (108.30*)	11.583
3	134.70 (132.55*)	10.659
4	162.23	9.217

* Calculated using Eq. (3).

Table 3.

	Stabilized deliverability coefficient	Turbulence coefficient	Exponent	Absolute open flow potential	
	a	b	n	Q_{abs}	R^2
	MPa²/(Nm³/min)	MPa²/(Nm³/min)ⁿ		Nm³/min	
Version A	0.6230	0.0001	2.4365	238.56	3.1055
Version B	0.6230	0.0001	2.4965	236.48	2.3251
Conventional method	0.5713	0.0013	2	241.55	3.4966

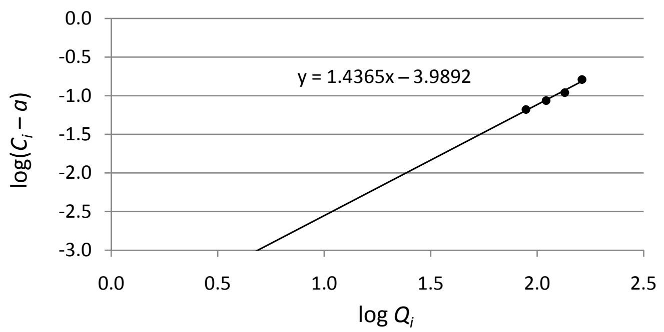

Fig. 1. $\log(C_i - a)$ vs. $\log Q_i$ curve

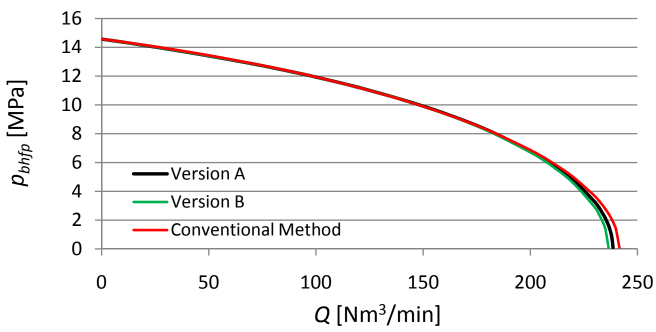

Fig. 2. Deliverability curves

Example no. 2

Well R-4.
Number of flow cycles $N = 4$.
Average pressure within drainage area $p_s = 25.98$ MPa.

Table 4.

Number of cycle	Gas flow rate Q_i [Nm³/min]	Stabilized sand face pressure p_{bhfpi} [MPa]
1	77.20	25.82
2	94.10 (93.58*)	25.78
3	115.60 (113.43*)	25.72
4	137.50	25.65

* Calculated using Eq. (3).

Table 5.

	Stabilized deliverability coefficient	Turbulence coefficient	Exponent	Absolute open flow potential	
	a	b	n	Q_{abs}	R^2
	MPa²/(Nm³/min)	MPa²/(Nm³/min)ⁿ		Nm³/min	
Version A	0.0933	0.000036	1.5709	1111.42	0.0297
Version B	0.0933	0.000037	1.5898	1119.21	0.0314
Conventional method	0.0848	0.000280	2	1411.57	0.0346

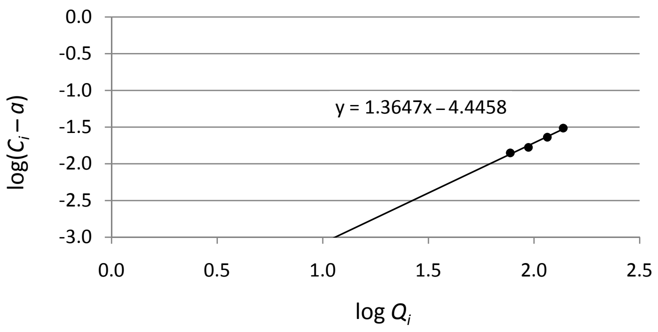

Fig. 3. $\log(C_i - a)$ vs. $\log Q_i$ curve

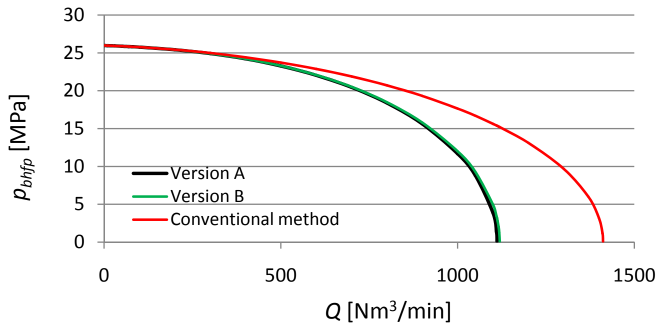

Fig. 4. Deliverability curves

Example no. 3

Well K-2.

Number of flow cycles $N = 4$.

Average pressure within drainage area $p_s = 18.58$ MPa.

Table 6.

Number of cycle	Gas flow rate Q_i [Nm^3/min]	Stabilized sand face pressure p_{bhfpi} [MPa]
1	27.40	17.65
2	42.40 (41.17*)	17.00
3	63.00 (64.91*)	15.84
4	99.90	13.14

* Calculated using Eq. (3).

Table 7.

	Stabilized deliverability coefficient	Turbulence coefficient	Exponent	Absolute open flow potential	
	a	b	n	Q_{abs}	R^2
	MPa^2/(Nm^3/min)	MPa^2/(Nm^3/min)^n	[–]	Nm^3/min	[–]
Version A	0.9686	0.0160	1.8388	163.14	0.9184
Version B	0.9686	0.0165	1.8304	163.71	1.0341
Conventional method	1.0414	0.0069	2	160.30	2.0431

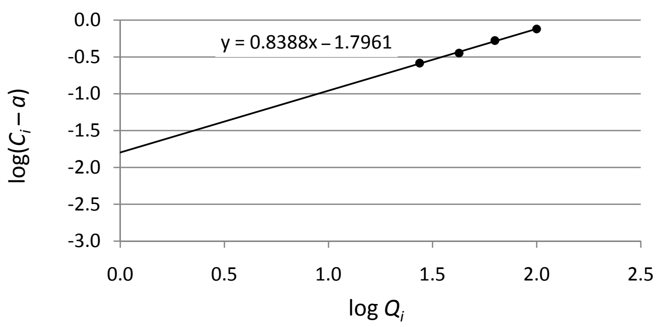

Fig. 5. $\log(C_i - a)$ vs. $\log Q_i$ curve

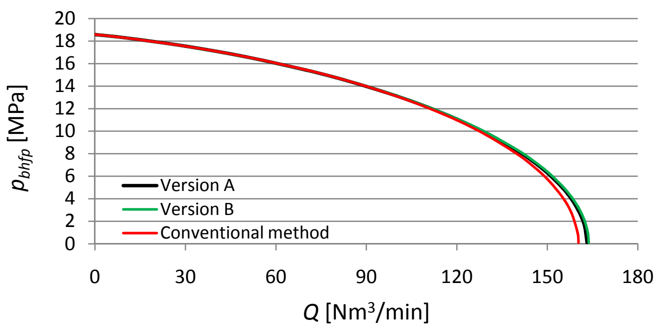

Fig. 6. Deliverability curves

Example no. 4

Well Z-7.

Number of flow cycles $N = 4$.

Average pressure within drainage area $p_s = 23.78$ MPa.

Table 8.

Number of cycle	Gas flow rate Q_i [Nm^3/min]	Stabilized sand face pressure p_{bhfpi} [MPa]
1	16.80	23.06
2	23.60 (24.00*)	22.42
3	34.80 (35.30*)	21.16
4	49.00	19.40

* Calculated using Eq. (3).

Table 9.

	Stabilized deliverability coefficient	Turbulence coefficient	Exponent	Absolute open flow potential	
	a	b	n	Q_{abs}	R^2
	$MPa^2/(Nm^3/min)$	$MPa^2/(Nm^3/min)^n$	[–]	Nm^3/min	[–]
Version A	0,1524	0,3115	1,6471	93,73	88,8697
Version B	0,1524	0,3116	1,6472	93,67	90,6460
Conventional method	1,2280	0,0564	2	89,86	100,2032

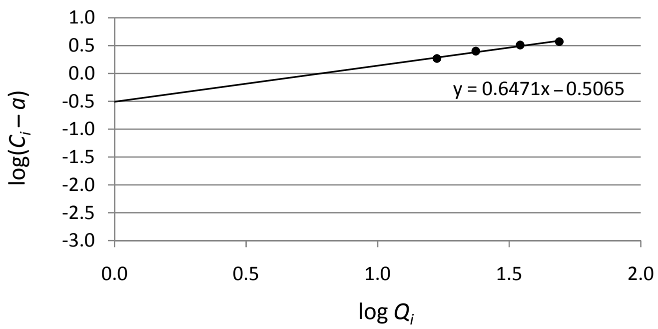

Fig. 7. log(C_i – a) vs. log Q_i curve

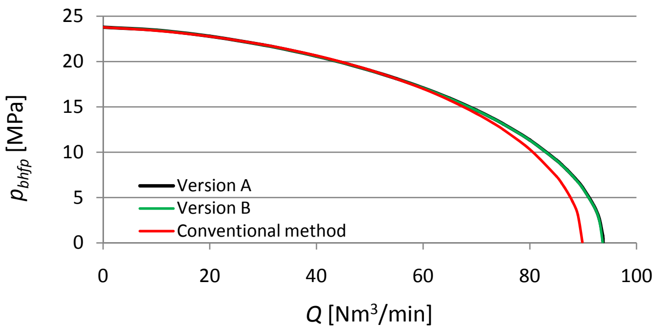

Fig. 8. Deliverability curves

Example no. 5

Well J-4.

Number of flow cycles $N = 4$.

Average pressure within drainage area $p_s = 25.613$ MPa.

Table 10.

Number of cycle	Gas flow rate Q_i [Nm^3/min]	Stabilized sand face pressure p_{bhfpi} [MPa]
1	45.90	25.601
2	57.50 (58.88*)	25.596
3	75.50 (75.54*)	25.587
4	96.90	25.574

* Calculated using Eq. (3).

Table 11.

	Stabilized deliverability coefficient	Turbulence coefficient	Exponent	Absolute open flow potential	
	a	b	n	Q_{abs}	R^2
	$MPa^2/(Nm^3/min)$	$MPa^2/(Nm^3/min)^n$	[–]	Nm^3/min	[–]
Version A	0.0057	0.00026	1.8826	2471.07	0.00001
Version B	0.0057	0.00026	1.8873	2449.43	0.00002
Conventional method	0.0070	0.00014	2	2133.74	0.00003

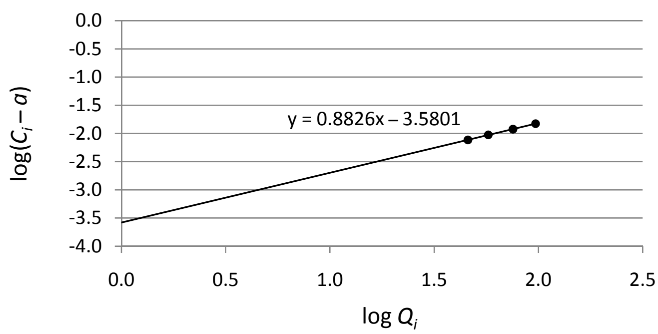

Fig. 9. log(C_i – a) vs. log Q_i curve

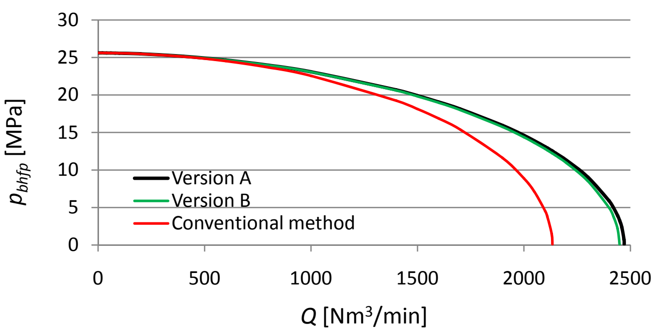

Fig. 10. Deliverability curves

Conclusions

1. In the case of gas wells with low to moderate flow rates the difference between results attained using the conventional and the proposed method is usually small. The IPR curves sometimes nearly coincide, and absolute open flow potentials do not differ much between the conventional and proposed method.
2. For high flow rate wells analyzed, the difference between the results of the conventional and proposed method becomes significant.
3. The fact that multiple datasets which satisfy the flow rate requirements given by Eq. 3 yielded reasonable results seems to support the usefulness of the proposed procedure.
4. Above conclusions are based on limited data (dozen or so wells were analyzed which is rather insufficient) and so the proposed testing procedure should be verified using much more data, specifically from high flow rate wells which are unfortunately unavailable.

Nomenclature

p_s – average pressure within drainage area of the well,
p_{bhfpi} – stabilized bottom hole flowing pressure (sand face flowing pressure) for *i*-th flow rate,
Q_1 – flow rate of the first cycle,
Q_N – flow rate of the *N*-th cycle,
Q_i – flow rate of the *i*-th cycle,
Q_{abs} – absolute open flow potential,
a, *b*, *n* – coefficients of Eq. 2,
i – index,
N – number of flow cycles.

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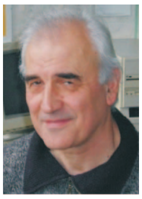
Literature

[1] Dake L. P.: *Fundamentals of reservoir engineering*. Elsevier 1978.
[2] *Energy Resources Conservation Board, Gas Well Testing*. Calgary 1978, third edition.
[3] Houperurt A.: *On the flow of gases in porous media*. Revue de L'Institute Francais du Petrole 1959, 14, 11, pp. 1468–1684.
[4] Johns L. G., Blount E. M., Glaze O. H.: *Use of the short term multiple rate flow test to predict performance of well having turbulence*. Society of Petroleum Engineers, Annual Fall Technical Conference and Exhibition, New Orleans, Louisiana 1976. http://dx.doi.org/10.2118/6133-MS.
[5] Johnson J. L., Lee W. J., Blasingname T. A.: *Estimating the stabilized deliverability of a gas well using the Rawlings and Schelhardt method: An analytical approach*. Society of Petroleum Engineers, Eastern Regional Meeting, Lexington, Kentucky 1991. http://dx.doi.org/10.2118/23440-MS.
[6] Lee J., Wattenbarger R. A.: *Gas reservoir engineering*. Society of Petroleum Engineers Textbook Series 1996, vol. 5.
[7] Szpunar T., Budak P.: *Uwagi na temat metodyki interpretacji danych wielocyklowego testu przypływu gazu do odwiertu*. Nafta-Gaz 2007, no. 2, pp. 115–124.



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Appendix A

If Eq. 2 is used for the interpretation of multi-rate gas well deliverability testing we can write for each stabilized flow rate Q_i

$$p_s^2 - p_{bhfp_i}^{\ 2} = aQ_i + bQ_i^n \qquad \text{(A.1)}$$

where $i = 1, 2, \ldots, N$; N – number of flow cycles.

We can write basing on Eq. (A.1):

$$\left(\frac{Q_i}{Q_{i+1}}\right)^{n-1} = \frac{C_i - a}{C_{i+1} - a} \qquad \text{(A.2)}$$

where $i = 1, 2, \ldots, N - 1$ and

$$C_i = \frac{{p_s}^2 - {p_{bhfp_i}}^2}{Q_i} \qquad \text{(A.3)}$$

We can see from (A.2) that if

$$Q_{i+1} = \left(Q_i Q_{i+2}\right)^{\frac{1}{2}} \qquad \text{(A.4)}$$

than

$$\frac{C_i - a}{C_{i+1} - a} = \frac{C_{i+1} - a}{C_{i+2} - a} \qquad \text{(A.5)}$$

where $i = 1, 2., \ldots, N - 2$; N – number of flow cycles.
Solving (A.5) for a we have:

$$a_i = \frac{C_i C_{i+2} - C_{i+1}^2}{C_i + C_{i+2} - 2C_{i+1}} \qquad \text{(A.6)}$$

On the right hand side of Eq. (A.6) there are measured values related to i-th, $i+1$ and $i+2$ flow cycles so we added index i to a. The most reliable value of a can be found as the minimum of $\sum_{i=1}^{N-2}(a - a_i)^2$ using the least squares method, i.e. calculating $\partial S/\partial a$ where

$$S = \sum_{i=1}^{N-2}\left(a - \frac{C_i C_{i+2} - C_{i+1}^2}{C_i + C_{i+2} - 2C_{i+1}}\right)^2 \qquad \text{(A.7)}$$

and so we have from (A.7)

$$a = \frac{1}{N-2}\sum_{i=1}^{N-2}\left(\frac{C_i C_{i+2} - C_{i+1}^2}{C_i + C_{i+2} - 2C_{i+1}}\right) \qquad \text{(A.8)}$$

If Eq. (A.4) is satisfied the flow rates which condition the validity of Eq. (A.8) can be calculated using the following formula

$$Q_{N-i} = Q_1^{\frac{i}{N-1}} Q_N^{\left(1 - \frac{i}{N-1}\right)} \qquad \text{(A.9)}$$

where $i = 1, 2, \ldots, N - 2$; N – number of flow cycles.

A) Knowing a the Eq. (A.1) can be presented in the following form:

$$\log(C_i - a) = (n - 1)\log Q_i + \log b \qquad \text{(A.10)}$$

If the $\log(C_i - a)$ vs. $\log Q_i$ data points are marked on the rectangular coordinates they should plot along the straight line – with slope $(n - 1)$ enabling calculation of n – which intersect the ordinate axis in $m = \log b$ for $Q = 1$ enabling calculation of b.

The n and b values may be also calculated using the following formulas:

$$n = 1 + \frac{N\sum_{i=1}^{N}\log(C_i - a)\log Q_i - \sum_{i=1}^{N}\log(C_i - a)\sum_{i=1}^{N}\log Q_i}{N\sum_{i=1}^{N}(\log Q_i)^2 - \left(\sum_{i=1}^{N}\log Q_i\right)^2} \qquad \text{(A.11)}$$

and

$$\log b = \frac{\sum_{i=1}^{N}\log(C_i - a)\sum_{i=1}^{N}(\log Q_i)^2 - \sum_{i=1}^{N}\log(C_i - a)\log Q_i\sum_{i=1}^{N}\log Q_i}{N\sum_{i=1}^{N}(\log Q_i)^2 - \left(\sum_{i=1}^{N}\log Q_i\right)^2} \qquad \text{(A.12)}$$

Equations (A.11) and (A.12) were derived using the least squares method i.e. solving the system of two Equations $\frac{\partial K}{\partial n} = \frac{\partial K}{\partial m} = 0$ where $m = \log b$ and

$$K = \sum_{i=1}^{N}\left[\log(C_i - a) - (n-1)\log Q_i - m\right]^2 \qquad \text{(A.13)}$$

B) If the requirements regarding gas flow rate of the series of gas flow cycles (Eq. (A.9)) enabling calculation of a are satisfied the n and b values may be calculated analytically in the following way:

Eq. (A.2) can be presented in a following form:

$$(n-1)\log\frac{Q_i}{Q_{i+1}} = \log\frac{C_i - a}{C_{i+1} - a} \qquad \text{(A.14)}$$

where $i = 1, 2, \ldots, N - 1$, and so

$$n_i = 1 + \frac{\log\frac{C_i - a}{C_{i+1} - a}}{\log\frac{Q_i}{Q_{i+1}}} \qquad \text{(A.15)}$$

On the right hand side of (A.15) there are measured values related to i-th and $i + 1$ cycle of gas flow so we added index i to the value of n. According to the least squares method the most reliable value of n is defined by:

$$n = 1 + \frac{1}{N-1}\sum_{i=1}^{N-1}\frac{\log\frac{C_i - a}{C_{i+1} - a}}{\log\frac{Q_i}{Q_{i+1}}} \qquad \text{(A.16)}$$

because it minimizes the sum

$$\sum_{i=1}^{N-1}(n - n_i)^2 \qquad \text{(A.17)}$$

Knowing value of n one can calculate b from Eq. (A.1)

$$b_i = \frac{C_i - a}{Q_i^{n-1}} \qquad \text{(A.18)}$$

where $i = 1, 2, \ldots, N$.

According to the least squares method the most reliable value of b is defined by:

$$b = \frac{1}{N}\sum_{i=1}^{N}\frac{C_i - a}{Q_i^{n-1}} \qquad \text{(A.19)}$$

because (A.19) minimizes the sum

$$\sum_{i=1}^{N}(b - b_i)^2 \qquad \text{(A.20)}$$