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Review of basic equations for evaluating drilling efficiency

Przegląd podstawowych równań oceny wydajności wiercenia

Rafail K. Mehtiyev, Yusif A. Tanriverdiyev

Azerbaijan State Oil and Industry University

ABSTRACT: The main goal of the reviewed article is to provide statistically determined relationships between the parameters of disintegration processes and the properties of rocks. The results of theoretical and experimental studies are discussed and analysed in the article. In relation to drilling, the formulas of drilling speed, depending on various parameters as an indicator that fully reflects the efficiency of the process, are given and compared. Thus, the drilling speed for percussion drilling is related to the characteristics of the rock and, at the same time, taking into account the constructional characteristics of the drilling tool. In percussion drilling, a new formula for drilling speed is presented, considering each impact and the frequency of the impact. The expression for the drilling speed was determined taking into account the degree of crushing of the rock matrix during drilling. Thus, the drilling speed is determined by considering the contact strength of the rock being drilled in the mechanical drilling method. The distribution of the stress state in the drilling zone was determined. Also, the shape and development characteristics of the cracks formed in the rock during the operation of the drilling tool (especially the dynamic percussions of the tool in the bottom zone of the well) were investigated. It should be noted that the energy intensity of the drilling process in the rock was determined by studying the next stages of the process of breaking the rock. The technical and economic indicators of the drilling works have been evaluated. Corresponding mathematical formulas are presented as a reliable calculation of drilling rates. The relevant mechanical and geophysical properties of the excavated rocks were considered. In the article, the drilling tools are selected depending on the drilling method, physical-mechanical properties of the rocks and geological conditions of the rock mass. The results of the obtained studies can be used in the design of the technological parameters of the drilling. The observations and results expressed in the article have a theoretical and practical aspect.

Key words: rock matrix, compressive stress, tensile stress, rock strength, drilling velocity, hardness coefficient of rock.

STRESZCZENIE: Głównym celem recenzowanego artykułu jest przedstawienie statystycznie określonych zależności między parametrami procesów rozwiercania a właściwościami skał. W artykule omówiono i przeanalizowano wyniki badań teoretycznych i eksperymentalnych. W odniesieniu do wiercenia podano i porównano wzory prędkości wiercenia w zależności od różnych parametrów jako wskaźnika w pełni odzwierciedlającego efektywność procesu. Zatem prędkość wiercenia przy wierceniu udarowym jest związana z charakterystyką skały i jednocześnie uwzględnia cechy konstrukcyjne narzędzia wiertniczego. Dla wiercenia udarowego przedstawiono nowy wzór na prędkość wiercenia uwzględniający każde uderzenie i częstotliwość uderzenia. Wyrażenie na prędkość wiercenia wyznaczono z uwzględnieniem stopnia zmiażdżenia matrycy skalnej podczas wiercenia. Tak więc prędkość wiercenia jest określana z uwzględnieniem wytrzymałości kontaktowej skały wierconej metodą wiercenia mechanicznego. Określono rozkład stanu naprężeń w strefie wiercenia. Zbadano również kształt i charakterystykę pęknięć powstających w skale podczas pracy narzędzia wiertniczego (zwłaszcza dynamicznych udarów narzędzia w strefie dennej otworu). Należy zaznaczyć, że energochłonność procesu wiercenia w skale została wyznaczona poprzez badanie kolejnych etapów procesu kruszenia skały. Tym samym dokonano oceny wskaźników techniczno-ekonomicznych prac wiertniczych. Odpowiednie wzory matematyczne przedstawiono jako rzetelny schemat obliczania postępu wiercenia. Uwzględniono odpowiednie właściwości mechaniczne i geofizyczne wydobytych skał. W artykule dokonano doboru narzędzi wiertniczych w zależności od metody wiercenia, właściwości fizyko-mechanicznych skał oraz warunków geologicznych górotworu. Wyniki uzyskanych badań mogą być wykorzystane w projektowaniu parametrów technologicznych prac wiertniczych. Przedstawione w artykule obserwacje i wyniki mają aspekt teoretyczny i aplikacyjny.

Słowa kluczowe: matryca skalna, naprężenie ściskające, naprężenie rozciągające, wytrzymałość skały, prędkość wiercenia, współczynnik twardości skały.

Corresponding author: Rafail K. Mehtiyev, e-mail: *rafail60mehtiyev@mail.ru*

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Introduction

In general, the collapse of a rock matrix occurs under the influence of a complex stress state, which is characterized by a combination of compressive, tensile and shear stresses. Objective information can be obtained by considering the physical laws of the collapsing process. However, the development of rigorous analytical calculation methods is always associated with some (sometimes controversial) assumptions and the popular idealization of the object rock, due to the complexity and uncertainty of the actual mechanism of rock collapse. This often causes the calculated indicators to differ significantly from their actual data. Nevertheless, the analytical approach is superior because it allows us to consider laws that operate objectively in nature.

The first theoretical scheme of impact fracture was proposed at the end of the 19th century by Dolezhalek (Czechoslovakia). Its further development is associated with the Russian scientist Uspensky (1924) (Samsuri, 2018). The theory is based on consideration of the system of forces when a wedge-shaped tool thrust into the rock. In this case, the physics of the destruction of rocks by the tool is not considered. Qualitatively, the percussion drilling mechanism can be represented as follows. With the dynamic introduction of the wedge, the initial volume of destruction is formed in the form of a groove, with a subsequent impact with the rotation of the tool at a certain angle, the secondary volume of destruction occurs. In the face plane, a shearing force T occurs, which destroys the rock in the volume of the sector between the furrows. Cyclically repeating, such a process leads to the destruction of the entire surface layer of the bottom of the hole or well to a certain depth.

Uspensky's theory gives the correct relationship between the force characteristics of the drill, but its use for special calculations is limited due to inherent disadvantages. First, it is the presence of empirical coefficients, the value of which cannot be determined from general physical concepts. In addition, the theory does not consider the physics of the disintegration process.

The Theoretical Part: Generally, the destruction of rocks occurs due to a complex stress state, which can be characterized by a combination of compressive, tensile, and shear stresses. Based on the assumption of the equality of the contribution of each type of stress in the process of destruction of rocks, Rzhevsky and Novik (2010) proposed a generalized indicator of the relative difficulty of destruction:

$$\Pi_{TP} = k_c C(\sigma_{sj} + \sigma_p + \tau_{sdv}) + B\gamma \tag{1}$$

where:

k_c – is the coefficient of structural weakening of the array,

σ_{sj}, σ_p, τ_{sdv} – strength of rocks during compression, tension, and shear,

γ – volumetric weight of rock.

$C = 5 \cdot 10^{-8}$ m and $B = 5 \cdot 10^{-5}$ m are the coefficients introduced for reasons of convenience in classifying rocks by destructibility. Based on formula (1), the following expression (2) can be set as a generalized indicator for different destruction methods, for drilling:

$$\Pi = C(\sigma_{sj} + \tau_{sdv} + B\gamma) \tag{2}$$

where $C = 7 \cdot 10^{-8}$ m and $B = 10^3$ m.

It is obvious that such generalized indicators, which do not take into account the real mechanism of destruction, the parameters of the technique and technology of the process, are applicable only for classification purposes for an enlarged comparative assessment of different rocks. More accurate information can be provided by statistically established relationships between the parameters of specific destruction processes and the properties of rocks.

Mechanical drilling methods can be divided into percussion and rotary (Latyshev, 2007; Guan et al., 2021). Percussion drilling occurs due to the short-term shock load of a wedge-shaped tool. The tool after the impact bounces off the bottom of the hole or well and rotates through a certain angle. Upon repeated impact, the formed sectors of the rock are chipped. The axial feed to the tool is either negligible or non-existent. During rotary drilling, the destruction of a hole or well at the bottom is carried out due to the cut of the rock when the cutter moves along a helix. This occurs because of a combination of rotational and translational movements, which are formed due to the application of significant torque and large axial forces to the drilling tool. There are no impact loads.

At present, a combination of percussion and rotary methods is widely used in drilling. In this case, shock loads are transferred to the tool continuously rotating under high axial pressure. Depending on the prevailing mechanism of destruction, percussion-rotational and rotational-percussion drilling methods are distinguished. In the first case, the main volume of destruction is formed due to impact. In rotary percussion drilling, the cutting mechanism is decisive, and the impact plays an auxiliary role. The boundary between these two methods is imprecise, and, in practice, the same drilling rig can implement both one and the other drilling mechanisms.

Of note is cone drilling. According to kinematics, this method is rotational. However, according to the mechanism of destruction, roller drilling is close to percussion or rotary percussion, depending on the type of bit used and the drilling mode. There are drilling methods (although not very common) that cannot be attributed to any of these types in their pure form. Such methods (for example, hydraulic, hydraulic vacuum or blast hole drilling) can be conditionally called special. The considered principles of classification are reflected in Table 1.

With regard to drilling, the indicator that most fully reflects the efficiency of the process can be the drilling speed V_b. Therefore, according to different authors (Rzhevsky and Novik, 2010; Baron, 1977; Vozdvizhensky et al., 1973), for perforating drilling, Vb is related to rock properties as follows:

$$V_b = \frac{k}{\sigma_{sj}^{0.59}} \tag{3}$$

where k is a coefficient that considers the design features of the drilling tool.

Considering the hardness coefficient of the rock, the following expression is given for the drilling speed V_b:

$$V_b = 415 - 32f + 0.65f^2 \tag{4}$$

where f is the coefficient of rock strength, V_b is mm/min.

Considering the formulas (3) and (4), we give the more efficient empirical expression for the drilling speed in the form of the following formula (5):

$$V_b = \frac{290N}{d^2(f_D + 2.6)} \tag{5}$$

where:
N – is the power of the perforator [kW],
d – is the diameter of the drilling bit [mm],
f_D – is the dynamic strength factor.

Table 1. Classification of rock drilling methods
Tabela 1. Klasyfikacja metod wiercenia skał

Types of drilling	Drilling methods	Main features
Mechanical	Kick (kick return)	Disruption by impact with subsequent rotation of the tool at the moment of return
	Impact-rotation	Disintegration by impact during the rotation of the tool
	Rotational impact	Disruption by shearing, mainly during impact loading on the tool
	Rotation	Cutting failure during tool rotation
	Sharoshkali	Disruption by puncture due to the insertion of an incisor
	Special	Hydraulic, hydraulic vacuum, electrohydraulic, blast hole drilling
Thermal	Thermal	Direct heat flow
	Electromagnetic	Heating in the electromagnetic field
	Thermomechanical	Combined action (effect) of direct or indirect heating with a mechanical tool

Considering all the expressions given above for the drilling speed V_b, we give the following more generalized expression (6):

$$V_b = \frac{k}{\sigma_{sj}\tau_{sdv} \cdot \text{tg}\left(\frac{\alpha}{2} + f_t\right)} \tag{6}$$

where:
α – is the angle of sharpening of the tool blade,
f_t – is the coefficient of friction between the tool and the rock.

With all this in mind, we present a new formula (7) for the drilling speed , considering each blow and the blow frequency in a perforating drill:

$$V_b = \frac{AnV_{\max}^{0.85}}{d_j} \tag{7}$$

where:
A – is the energy of a single impact C,
n – is the frequency of beats per minute,
d_j – hole diameter [mm].

Considering the degree of rock crushing during drilling, the following expression (8) can be given for the drilling speed V_b:

$$V_b = \frac{0.003AnV_{\max}}{d^2} \tag{8}$$

where $V_{\max}$ is the crushability index (according to Baron (1977)).

One of the interesting reasons is to determine the drilling speed by considering the contact strength of the drilled rock in the mechanical drilling method. For this reason, we give the following expression (9) for the drilling speed, taking into account the contact strength of the rock:

$$V_b = \frac{1}{7.2 \cdot 10^{-9} P_k - 2.75} \tag{9}$$

where P_k is the contact strength of rock.

So, let's mention below a short analysis of empirical formulas (3) – (9). No smaller number of such statistical equations are known for rotary, cone and other drilling methods. An analysis of these equations shows that the calculated drilling speeds for the same rocks and conditions can differ by several times. Attention is also drawn to the different nature of dependencies. All this becomes clear if we consider that the equations were found from experimental data for specific rocks, types of perforators, drilling technologies, including working conditions, qualifications of drillers, etc. Therefore, each equation can be used only for those conditions for which it was found.

During drilling, during the action of its P_y force along the Y axis, the blade of the drilling tool is introduced into the rock to a depth h. In this time, it is necessary to overcome the force of resistance of the rock to collapse F_{sm} and the friction force F_{tr}. Then the failure condition can be written as formula (10):

$$F_Y = F_{sm} + F_{tr} \quad (10)$$

When the wedge is introduced, reactions of repulse of the rock M occur, which are perpendicular to the generators of the wedge. The vector sum of these forces F_{sm} is given in the expression (11) below:

$$F_{sm} = 2M \sin\left(\frac{\alpha}{2}\right) \quad (11)$$

The strength of rock matrix is determined by the following (12) expressions:

$$\sigma_{sm} = F_{sm} / S_{sm} \quad \text{or}$$
$$F_{sm} = \sigma_{sm} S_{sm} \quad (12)$$

Uspensky's theory (1924) gives the correct relationship between the power characteristics of drilling, but its use for specific calculations is limited due to its inherent shortcomings. Firstly, this is the presence of empirical coefficients, the value of which cannot be determined from general physical concepts. In addition, the theory does not consider the physics of the destruction process. In particular, the values of crushing and shearing strength of rocks used in the calculation formulas do not have a clear physical meaning. However, the reliability of the calculation scheme and the correctness of the derivation of the main relationships make it possible to use this theory as a base in the development of methods for designing drilling processes.

To understand the physics of drilling, it is necessary to consider the distribution of stresses during the placement of the tool into the rock. In the simplest case, the action of a concentrated force P on an elastic half-space can serve as a process model. The analysis of the stress state in such a model is based on the solution of the Boussinesq spatial problem (Spivak, 1967; Lin et al., 2019). Consider an arbitrary point A at some distance R from the point of application of the concentrated load. The full stress vector σ_R coincides in direction with the beam R and is directed to the point of application of the load 0 at an angle β. If a sphere with a diameter d is drawn through the origin (point 0) and point A, then, for all points of this sphere the total stresses will be the same and that stress is determined by the following formula (13).

$$\sigma_R = \frac{3P}{2\pi d^2} \quad (13)$$

Such a sphere of equal stresses in a plane is transformed into a circle. The total stress can be reduced to normal (to the loading surface) and tangential components, and the following formulas (14), (15) are determined:

$$\sigma_z = \frac{3P}{2\pi d^2} \cdot \cos\beta \quad (14)$$

$$\tau_{xz} = \frac{3P}{2\pi d^2} \cdot \sin\beta \quad (15)$$

Thus, along the axis of symmetry, all normal stresses are compressive, i.e., the rock is in volumetric compression. In real conditions, the drilling tool has a certain shape and geometric dimensions, so the considered model (concentrated load condition) is not used. Depending on the dimensions of the geometry of the drilling tool, the loading platform can have a variety of sizes and shapes. In theory, one of the following design schemes is usually adopted: a cylindrical stamp, a sphere, or a wedge. With a difference in quantitative assessment, the qualitative patterns of stress distribution under the drilling tool remain unchanged. Therefore, it suffices to consider the model of penetration into the rock of a cylindrical stamp with a flat base.

The distribution of pressure over the plane of contact of the flat base of a cylindrical punch with a radius a with the rock is not uniform and depends on the distance x from the punch axis (Spivak, 1967):

$$\sigma(x) = \frac{P}{2\pi a\sqrt{a^2 - x^2}} \quad (16)$$

Examining expression (16), it is clear that, it follows that on the axis of the stamp (at $x = 0$) the pressure will be the smallest: $\sigma(x = 0) = P/2\pi a^2$. In the contact contour (at $x = a$), the pressure becomes infinitely large: $\sigma(x = a) \to \infty$.

The theory and experimental studies show that, on the contact surface of the stamp with the rock, the vertical and horizontal stresses are maximum and equal to each other: i.e. $\sigma_z = \sigma_x = \sigma_y = \max$, and there are no shear stresses: $\tau = 0$. Therefore, a thin near-surface layer of the rock is in uniform all-round compression, i.e., cannot collapse. However, as we move away from the contact surface ($Z > 0$), the normal stresses decrease. Moreover, horizontal stresses ($\sigma_x = \sigma_y$) decrease more intensively than vertical σ_z. The difference in normal stresses (according to Mohr's theory) causes shear stresses to appear. With an increase in this difference, the shear stresses increase, reaching a maximum at a depth approximately corresponding to the punch radius a. It is these tangential stresses that determine the destruction of the rock under the stamp.

Based on this model, Ostroushko (1966) developed a theory of drilling, the main features of which can be represented as follows (Nguyen et al., 2016). The destruction of the rock under the stamp is cyclically dampened. Each cycle can be divided into several stages. At the first stage, as a result of the action of the load P_Y along the Y axis, an elastic deflection of the rock occurs under the stamp. The deformation of the rock in this case corresponds to Hooke's law. At the second stage, when the stresses of the rock under the stamp reach the elastic limit, irreversible changes occur in it, which are as follows. In areas at an angle of 45°, shear stresses reach a maximum, and a system of cracks is formed. Rock deformation becomes non-linear. In the third stage, a cone-shaped core is formed under the tool, which is limited by compacted cracks. Rocks in the core are in volumetric compression. Expanding under the action of the load, the compaction core pushes the rock along the cracks. After that, the core is instantly unloaded and the volumetric stress state becomes uniaxial. The elastic energy stored in the compaction core is spent on the destruction and regrinding of the rock. The load drops sharply, and the stamp plunges into the rock to a depth h_0. At the same time, crushed rock remains under the stamp at the base of the destruction cone. At the fourth stage, when the tool moves, the destroyed rock is compacted under the stamp, which is accompanied by an increase of the load P_Y along the Y axis. In this case, the compacted rock serves as an additional working fluid that transfers the load to the surrounding massif. Then, the destruction cycle is repeated, but with a greater axial force, since additional energy is spent on compacting the destroyed rock under the stamp and overcoming the forces of friction of the side surface of the stamp against the rock. The number of destruction cycles depends on the magnitude of the axial force and the properties of the rocks. In this case, from cycle to cycle, the resistance to the introduction of the tool increases, and the amount of destruction, as well as the amount of deformation, decreases.

Knowing the depth and volume of destruction, it is possible to theoretically determine the load P_Y along the Y axis required for the effective destruction of rocks, the energy intensity of destruction, the drilling speed, the parameters of the drilling tool, etc. However, these calculations are inappropriate for the following reasons. This calculation was performed for the first failure cycle. However, as Ostroushko (1966) himself notes, the volume of destruction in subsequent cycles decreases as the tool deepens into the massif and does not have regular geometric shapes, being outlined by complex surfaces that depend on the properties and real structure of rocks and are largely random. In accordance with the law of distribution of stresses under the drilling tool (see equation (7)), the maximum shear stresses occur at a depth equal to half the diameter of the die $d/2$. Then, the opening angle of the seal cone $\alpha = 45°$. However, in real drilling $\alpha = (60–75)°$. Moreover, the actual shape of the seal core is generally not conical. Nevertheless, the theory of Ostroushko (1966) correctly reflects the cyclic nature of the destruction of rocks during drilling. Particularly fruitful is the idea of the formation of a compaction core and its role in the subsequent spalling of the rock. This is proved by numerous experiments and practice data.

According to Shreiner (1950) and other researchers (Schreiner, 1950; Spivak, 1967; Hu et al., 2022), the seal core under the stamp has a spherical geometry. The rocks in this volume are in volumetric compression. Special studies of the patterns of destruction of the rock under the stamp show the following. Due to rock overstress, elliptical fracture zones are formed along the contact contour, from which a system of vertical cracks branches off. These cracks limit the zone of weakening; the rocks in which are broken by a network of cracks and have a reduced strength. This zone is called the pre-fracture zone. For the formation of this zone, additional energy is expended, but this is compensated by the subsequent facilitation of rock destruction during repeated impacts.

Dispersed samples of rock matrix under the drilling tool considered above do not take into account the time factor, that is, the process is described statically. The patterns of rock destruction under the drilling tool considered above do not take into account the time factor, i.e., the process is described statically. In real percussion and percussion-rotary drilling, the time of penetration of the tool to a depth of 3–5 cm is 200–400 ms. During this time, the initial shock pulse at an average velocity of elastic waves in the rock of 2000–4000 m/s propagates to a depth of 80–160 cm (Kutuzov, 1972; Wei et al., 2016). In this case, the magnitude of stresses in the rock decreases exponentially with distance.

The dynamics of the tool penetration into the rock under impact loads can be represented as a set of the following stages (Latyshev, 2007; Zhang and Yin, 2017):

1. Surface destruction. When the tool moves, the protrusions and roughness formed at the moment of the previous impact are crushed. Destruction products, falling under the tool, ensure its tight contact with the rock.
2. Formation of a pre-fracture zone. Due to the impact of the tool with the rock and the instantaneous transfer of the elastic impulse, radial cracks are formed. Propagating deep into the massif at a distance much greater than the depth of the puncture zone, these cracks form a zone of weakening of the rock.
3. Volume loosening. With further introduction of the tool, the normal stresses in the rock will increase until their critical values, equal to the strength during volumetric compression of the rock, extend to a layer with a thickness equal to the average size of the grains that form this rock. This is due to the fact that the strength of the contacts is always less than the strength of the crystals themselves. In this case, fracture along contacts separates this layer into separate crystalline fragments, and a layer of bulk fracture is formed.

4. Chip rock. With further movement of the tool, the magnitude of shear stresses in the rock increases. The loosening of the rock in the layer of volumetric destruction contributes to an increase in these stresses. When the tangential stresses reach a critical value, chipping of the rock occurs. In a real process, fracture is determined by a combination of cleavage mechanisms. After this, the stresses in the rock drop sharply, and the cycle of destruction is repeated.

As shown by Shreiner (1950), the destruction of rocks by a mechanical tool is always associated with its penetration into the surface of the rock. In this case, the process can be represented as an indentation of a flat stamp into a semi-infinite body (an elastic or elastic-plastic half-space). Due to the acting load, an area of volumetric compression is formed under the stamp, called the compaction core. A fruitful idea about the formation and role of the compaction core in the process of destruction of rocks was expressed by Ostroushko (1966). In general, the longitudinally compressed core of the seal expands in the transverse direction. The tensile stresses arising near this core lead to the destruction of the rock. The destruction of the rock under the tool occurs as a result of a combination of cleavage (destruction to the second free surface formed as a result of the previous act of destruction) and puncture (destruction to the same surface on which the destructive force is applied).

In the simplest case of a cleavage onto a second free surface, Protasov (1985) proposed the following model for the destruction of rocks. When the force F of the tool (stamp) is introduced into the rock by the value Δh, a compaction core with a volume V_0 is formed under it, where the rock is in a state of all-round compression. On the lateral surface of the compaction core, an array reaction P occurs, which depends on the force F and the distance to the free surface H. The destruction of the rock by chipping occurs along the site S_0, and the volume of destruction is V.

The compaction core, expanding under the action of the force F to the free surface, performs the work of separating the volume V from the array. That work is calculated using the following formula (17):

$$A = \frac{kV\sigma_p^2}{E} \tag{17}$$

where k is a coefficient that takes into account the difference between the real behaviour of the massif and the ideally elastic one; can be interpreted as a coefficient of plasticity.

On the other hand, the work of the compression core is defined by the expression (18) of its volume increase:

$$dV = dV_F - dV_p \tag{18}$$

where dV_F and dV_p are the increase in the volume of the seal core, respectively, from the force F and the rebound reaction p.

From the energy conservation law, for a given fracture scheme, the energy balance equation (19) can be written (Protasov, 1985; Mazen, 2021):

$$\frac{kV\sigma_p^2}{E} = \frac{2\sigma_p V\nu F}{HBE} - \frac{3\sigma_p^2 V^2 b^2 \sigma_0 A_1(1-2\nu)}{2H^2 FE} \tag{19}$$

where ν is the Poisson's ratio of the rock; A_1 and B are the width and length of the tool blade; σ_0 is the strength of the rock under all-round compression; can be taken as $\sigma_0 = 0.1E$; b is the volume shape factor V; for a rectangular figure $b = 1$.

From the equation (19), the value of the volume of crushed rock can be determined by the formula (20):

$$V = \frac{2H^2 F}{2\sigma_p b\sigma_0 A_1(1-2\nu)}\left(\frac{2\nu F}{HB} - k\sigma_p\right) \tag{20}$$

It is obvious that destruction will occur if $V > 0$, then the minimum required force on the tool, leading to a rock chip, is determined by the expression (21):

$$F = \frac{k\sigma_{pHB}}{2\nu} \tag{21}$$

In turn, the force on the tool is determined by the corresponding energy Q and the impact energy is given in expression (22).

$$F = \frac{2QE}{A_1\sigma_0} \tag{22}$$

Considering that $\sigma_0 = F/(A_1 B)$, we get:

$$F = \sqrt{2QEB} \tag{23}$$

and

$$\sigma_0 = \sqrt{\frac{2QE}{A_1^2 B}} \tag{24}$$

The specific energy consumption of breaking $q = Q/V$, then, taking into account equations (20), (23), (24), we get the expression (25):

$$q = \frac{3k\sigma_p^2 b(1-2\nu)}{2E\nu^2} \tag{25}$$

It can be seen from expression (20) that the destroyed volume of the rock depends nonlinearly on the value of H, often called the chip thickness. Then the extremum of the function at $dV/dH = 0$ will correspond to the optimal chip thickness (formulas (26), (27)):

$$H_{opt} = \frac{\nu F}{k\sigma_p B} \tag{26}$$

or with (23) taken into account.

$$H_{opt} = \frac{2bv\sqrt{QE}}{k\sigma_p\sqrt{2b}} \tag{27}$$

As shown in (Protasov, 1985; Huang et al., 2017) by replacing the force F in formula (20) with the energy Q, the volume of a single the phenomenon of dispersion of a unit volume can be determined by the following formula (28):

$$V = \frac{4v^2QE}{3k\sigma_p^2 b(1-2v)} \tag{28}$$

It is known from the theory of elasticity that the efficiency of the disintegration process is determined by formula (29):

$$\eta = \frac{2v^2}{3(1-2v)} \tag{29}$$

The share of energy directly spent on the destruction of the rock in the total energy transmitted to the tool.

Then, the disintegrated volume of rock V is determined by the expression (30).

$$V = \frac{2QE\eta}{k\sigma_p^2 b} \tag{30}$$

The linear rate of destruction is defined as $V_b = V/St$, where t is the time of a single act of destruction. As applied to the percussion drilling process, $S = (\pi d^2)/4 = (\pi B^2)/4$ and $t = 1/n$, where n is the impact frequency. Hence the drilling speed:

$$V_b = \frac{4Vn}{\pi B^2} \tag{31}$$

Application of the formula (31) and the loading of the massif bounded by the free surface with the limited area S by the force F causes the local disintegration of the rock matrix. According to the scheme proposed by Protasov (1985), a primary compaction core V_{01} is formed under the tool, the deformation of which in the transverse direction ΔV_1 causes a rock response. In this case, a secondary compaction core V_{02} is formed, which, expanding in the direction of the free surface, pushes the rock volume V out of the massif.

Guided by the same logic as in the case of a cleavage, Protasov (1985) obtained the following energy balance equation for a cleavage:

$$\frac{k\sigma_p V}{E} = \frac{2v^2F^3\eta}{3A_1B^2E(1-2v)} \tag{32}$$

From equation (32), the collapsing event V is determined by the following formula (33):

$$V = \frac{8H^2F}{3\sigma_p b^2\sigma_0 A_1(1-2v)}\left(\frac{vFb\eta}{HB} - k\sigma_p\right) \tag{33}$$

From here at $V = 0$:

$$F = \frac{k\sigma_p HB}{vb\eta} \tag{34}$$

The specific energy intensity of collapsing is calculated by the following formula (35):

$$q = \frac{3k\sigma_p b(1-2v)}{4Ev^2\eta^2} = \frac{k\sigma_p b}{2E\eta^3} \tag{35}$$

Function (33) also has an extremum at $dV/dH = 0$. Hence, the optimal depth of fracture by a puncture is determined by the expression (36):

$$H_{opt} = \frac{vFb\eta}{2k\sigma_p B} \tag{36}$$

Similarly to equations (28)–(30) for a pinhole, we can write expression (37):

$$V = \frac{16Q^3E^3v^2\eta^2}{3A_1^4B^2\left((1-2v)k\sigma_p b^2\sigma_0^4\right)} = \frac{8Q^3E^3\eta^3}{A_1^4b^2B^2k\sigma_p^2\sigma_0^4} \tag{37}$$

The maximum contribution of the cleavage mechanism will be occur when the distance from the destructive tool to the second exposed plane corresponds to H_{opt} (equation (26)). With an increase in this distance, the share of the cleavage mechanism decreases and, at a certain critical distance, H_{kr}, is reduced to zero, i.e., the destruction of the rock is possible only by cleavage. The value of H_{kr} can be determined from equation (20) at $V = 0$. In the general case, the value of H_{kr} is determined by expression (38).

$$H_{kp} = \frac{2vF}{k\sigma_p B} \tag{38}$$

or taking into account (23):

$$H_{kp} = \frac{2v\sqrt{2QE}}{\sqrt{k\sigma_p B}} \tag{39}$$

Comparing expressions (26) and (39), it is easy to see that H_{kp} is 2 times greater than H_{opt}.

Conclusions

Analysing the obtained equations, we can draw the following conclusions: The structure of the relationships for both the cleavage and the puncture is of the same type. The main difference lies in the different degree of the exponent η. The real process of destruction of rock by a mechanical tool always contains elements of chipping. However, the specific energy capacity of a puncture is $1/\eta^2$ times greater than that of a cleavage. In this regard, the parameters of the destruction process should be selected in such a way that the largest possible volume of rock is chipped off. In particular, the strength values used in the calculation formulas do not have a clear physical meaning of splitting and tearing of rock matrix. However, the reliability of the calculation scheme and the correctness of the extraction of the main relationships allow this theory to be used as a basis in the development of methods of designing drilling processes.

Thus, a unified working theory of drilling has not yet been created. However, numerous studies of the theory and practice of the process allow the optimal drilling equipment for the given mine-geological conditions to be chosen and effective modes of its operation to be selected.

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Rafail Karim MEHTIYEV, Ph.D.
Assistant Professor at the Department of General and Applied Mathematics
Azerbaijan State Oil and Industry University
16/21 Azadliq Ave, AZ1010 Baku, Azerbaijan
E-mail: *rafail60mehtiyev@mail.ru*

Yusif Alovsat TANRIVERDIYEV, M.Sc.
Teacher at the Department of Material Science and Processing Technology
Azerbaijan State Oil and Industry University
16/21 Azadliq Ave, AZ1010 Baku, Azerbaijan
E-mail: *yusif.tanriverdiyev@asoiu.edu.az*