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Methods for solving two-dimensional tasks of cutting raw materials

Metody rozwiązywania problemów związanych z cięciem materiałów w dwóch wymiarach

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ABSTRACT: The article discusses the methodology for solving two-dimensional material cutting problems, widely used in practice and applied to industrial equipment. Several modifications of the original problem are considered. An interactive optimization procedure is presented for a general two-dimensional material cutting problem. When cutting correctly, the two dimensions of the cut pieces (usually length and width) must be consistent with the length and width of the sheet. One of the problems most frequently encountered in literature and in practice is the problem of cutting a rectangular material into rectangular pieces. Therefore, this work focuses on this task. First, a two-dimensional problem of cutting material is formulated. Next, methods for solving the problem of cutting material and the related problem of constructing (creating) a template is outlined. The solution method includes a new interactive (dialogue) optimization procedure. A very interesting feature of 2D problems is that there are different options that arise from practical requirements due to the type of material and manufacturing process constraints. A description of the general technique would be incomplete without mentioning how it can be modified to apply specific practical problems. Therefore, the paper briefly discusses some practical applications and describes ways to modify the general methodology to solve these practical problems.

Key words: guillotine cut, vector, interactive optimization, design, vertical cut, modification.

STRESZCZENIE: W artykule omówiono metodologię rozwiązywania dwuwymiarowych problemów cięcia materiałów, szeroko stosowaną w praktyce i wykorzystywaną w urządzeniach przemysłowych. Rozważono różne modyfikacje pierwotnego problemu. Opracowano interaktywną procedurę optymalizacji dla ogólnego problemu dwuwymiarowego krojenia materiału. Podczas prawidłowego krojenia, dwa wymiary wyciętych elementów (zwykle długość i szerokość) muszą być zgodne z długością i szerokością arkusza. Jednym z najczęściej spotykanych w literaturze i praktyce problemów jest cięcie materiału prostokątnego na elementy prostokątne. Dlatego w niniejszej pracy skoncentrowano się na tym zadaniu. Przede wszystkim sformułowany został dwuwymiarowy problem materiału, a następnie omówiono metody rozwiązywania problemu krojenia materiału oraz pokrewnego problemu konstruowania szablonu. Metoda rozwiązania obejmuje nową interaktywną procedurę optymalizacji tego procesu. Bardzo interesującą cechą problemów 2D jest to, że istnieją różne opcje wynikające z wymagań praktycznych, podyktowanych rodzajem materiału i ograniczeniami procesu produkcyjnego. Opis ogólnej techniki byłby niepełny bez wzmianki o możliwości jej modyfikacji w celu zastosowania do konkretnych problemów praktycznych. Dlatego w artykule omówiono pokrótce niektóre zastosowania praktyczne i opisano sposoby modyfikacji ogólnej metodologii w celu rozwiązania tych problemów.

Słowa kluczowe: pcięcie gilotynowe, wektor, optymalizacja interaktywna, projektowanie, cięcie pionowe, modyfikacja.

Introduction

Two-dimensional tasks involving cutting materials (raw materials) appear in manufacturing processes where the material in the form of a sheet must be cut into smaller parts (pieces). When cutting correctly, the two dimensions of the cut pieces (usually length and width) must be consistent with the length and width of the sheet. This is in contrast to a one-dimensional cutting problem, where typically a strip of material (i.e., its length) is cut into smaller lengths. A review of methods for solving a one-dimensional problem is available in (Golden, 1976). The two-dimensional cutting problem has numerous applications: cutting and/or stamping of sheet metal, cutting of sheets of glass, paper, fabric, leather, film, plastic, etc. This task can also have applications in loading and packaging. Loading trucks, freight railway cars, and laying stacks are tasks equivalent to the two-dimensional problem of cutting material. In these problems, two-dimensional templates are

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determined and the load is stacked (analogous to cutting) according to these templates.

The two-dimensional problem of cutting raw materials is to determine a method for obtaining a given number of rectangular pieces with dimensions (l_i, ω_i), $i = 1,\dots, n$, from an unlimited rectangular material with the size of (L, W). In this case, the goal is usually to minimize the total waste (clippings). However, practice also provides other objective functions: a) maximizing the total cost of the pieces (where the cost depends on the size and quality of the piece); b) minimizing the time (or cost) of setting up a machine to implement templates.

When formulating the problem, the following notes are introduced:

Q – matrix of valid templates (column Q corresponds to a template); template ($q_1, q_2,\dots, q_m$) gives q_1 pieces with dimensions (l_1, ω_1), q_2 pieces with dimensions (l_2, ω_2),..., q_m pieces with dimensions (l_m, ω_m);

N – ($m \cdot 1$) – vector of needs; N_i – need for pieces with dimensions (l_i, ω_i);

X – ($m \cdot 1$) – vector; x_i – number of templates of the j-th type cut from the material;

$a_i = (l_1 \cdot \omega_i)$ – area of the i-th piece, $i = 1,\dots, m$.

Statement of a two-dimensional problem of cutting raw materials

In the problem of minimizing the total amount of waste, it is required to determine x_i, $i = 1,\dots, m$, so that,

$$\min \sum_{j=1}^{m} x_i$$

$$QX = N,\ x_i \geq 0 \text{ – whole number} \qquad \text{(A1)}$$

This formulation is similar to the one-dimensional cutting problem. The main feature of this task is to create templates for cutting, denoted ($q_1,\dots, q_m$). In one-dimensional problems, these templates are easily obtained by solving the corresponding loading (knapsack) problem (Golden, 1976). Since in this case one size is given, solutions to the backpack problem are always obtained within the boundaries of the material. However, such a solution becomes relatively difficult in the two-dimensional case, since the solution to the corresponding loading problem may not be within the boundaries of the material. The condition that the template fits within the boundaries of the material is also called the admissibility condition. Because of this condition, the technique for creating valid templates in the two-dimensional case will no longer be so simple. Therefore, before presenting in methods for solving the problem (A1), we will consider techniques for creating valid templates.

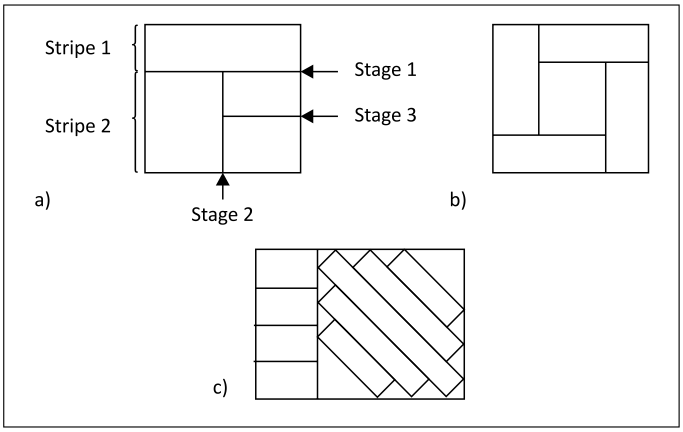

Figure 1. Types of cuts; a) guillotine cut; b) not a guillotine cut; c) non-orthogonal section

Rysunek 1. Rodzaje cięć; a) cięcie gilotynowe; b) cięcie niegilotynowe; c) przekrój nieortogonalny

The task of creating a template

If v_i is the cost of the i-th type of piece, then the problem of creating a template is formulated as follows:

$$\max(v_1 q_1 + \dots + v_m q_m)$$

providing

$$a_1 q_1 + \dots + a_m q_m \leq A = L \times W,\ q_1,\dots, q_m \qquad \text{(B1)}$$

integers giving a valid template.

The cost of a piece can be the market value of this piece, the cost of cutting this piece, or it is a characteristic of the quality of this piece, if the quality of the piece depends on its location on the workpiece of the material. The difficulty in solving problem B1 lies in the fact that the condition for the admissibility of a template is difficult to describe mathematically, and without this condition, the templates obtained by solving (B1) will not necessarily be admissible.

When creating a template, an important factor to consider is the type of cut used in cutting the template from the material. One type of cut can be from one edge of the rectangular material to the other, called a “guillotine” cut. The material is cut with a guillotine sequentially in separate stages (operations), as can be seen in Figure 1a. This type of cut occurs when cutting is done on presses. Figure 1a shows a more general type of cutting, where the cut is not made from edge to edge. The sections shown in Figure 1a and c, are performed parallel to one of the edges of the workpiece. These types of cuts are called orthogonal. The sign of a non-orthogonal section is shown in Figure 1c. Non-guillotine cutting is possible when it is performed using a torch, saw, or laser. Non-guillotine arrangement of pieces occurs, in loading and packing problems.

Now a technique for solving the problem of creating templates based on guillotine cuts will be described. At the end, the solution procedure for the general type of cuts will also be given. Problem B1 for the case of guillotine cuts will be designated as problem (B2).

Two-step method

Karimov and Abbasov (2016) proposed a two-stage solution procedure (B2). According to this procedure, the knapsack problem for (B1) is solved at each stage, to generate feasible templates. The required pieces are obtained by cutting the material in steps with a guillotine. In a two-step procedure, in Step 1 the material is cut into strips, and in Step 2 the strips are cut into the required pieces (if required final trimming, can be done later). At each step, corresponding tasks about the backpack are set. At step 1, for each width ωi, valid templates are determined that fit rectangles of width $\omega_j \leq \omega_i$ in strips of size $\omega_i \times L$. The following backpack problem is solved to do this:

$$v_1^* = \max\left(v_1 q_1 + \ldots + v_t q_t\right)$$

providing:

$$l_1 q_1 + \ldots + l_t q_t \leq L \qquad \text{(B2)}$$

where: $l_1,\ldots l_t$ and $q_1,\ldots q_t$ are the length and number of pieces, respectively, for which $\omega_j \leq \omega_i, j = 1,\ldots, t$. In step 2, the knapsack problem is to fit strips with the size of $\omega_i \cdot L$ into a rectangular material of the size $W \times L$ so that

$$\max(v_1^* q_1 + \ldots + v_m^* q_m)$$

providing:

$$\omega_1 q_1 + \ldots + \omega_m q_m \leq W$$

There are several methods for solving the one-dimensional knapsack problem (Golden, 1976).

Dynamic programming

Dynamic programming (DP) has been quite widely used to solve template creation problems. In the publication of Vahidov et al. (2008) a procedure for solving (B2) was proposed, in which there is no restriction on the number of cut rectangles of the same type. If vi is the cost of the i-th piece, then the recursive DP equations for two-step cutting are written as follows:

$$F_0(x, y) = \max\{0, v_i, l_i \leq x, \omega_i \leq y\},$$

$$F_1(x, y) =$$

$$= \max\{F(x, y-1), F(x_1, y) + F(x_2, y); x \geq x_1 + x_2, 0 < x_1 \leq x_2\}$$

$$F_2(x, y) =$$

$$= \max\{F(x-1, y), F(x, y_1) + F(x, y_2); y \geq y + y_2, 0 < y_1 \leq y_2\}$$

where: $F_1(x, 0) = 0$, $F_2(0, y) = 0$, $F(x, y) = \max\{F_0(x, y),$
$F_1(x, y), F_2(x, y)\}$.

Here, it is assumed that l_i and ω_i are integers. The solution to this equation is quite simple and carried out as follows. First, set $x = y = 1$, changing these values separately. For any values of x_2 and y_2, the value of x_1, starting from $x_1 = 1$, increases by one each time. In this case, y_2 remains constant and the maximum is determined from $F(x_1, y_2) + F(x_2, y_2)$ and $F(x_1 + x_2, y_2)$. This procedure continues until x_1 exceeds x_2 or $x_1 + x_2$ becomes greater than L, where the current value of x_0 is fixed. Then, with a fixed current x_2, the value of y_1, starting from $y_1 = 1$, increases each time by one and the maximum is determined from $F(x_2, y_1) + F(x_2, y_2)$ and $F(x_2, y_1 + y_2)$. This continues until $y_1 > y_2$ or $y_1 + y_2 > W$ is completed. Then x_2 is incremented by one and the previous process is repeated with y_2 fixed until $x_2 > L$ is completed. After this, y_2 is increased by one, and the whole process is repeated, starting with $x_2 = 1$. The process stops when y_2 exceeds W.

Recursive method

A recursive method for solving the problem (B2) was proposed by Hertz (Aliyeva et al., 2021). It is based on the following recursive property: either the large rectangle is already one of the required rectangles, or the first cut line produces two rectangles, each cut optimally. This property means that every possible first cut of a large rectangle R should be tried. However, it is easy to show that a) only those cuts of R that are integer multiples of the lengths or widths of the pieces should be considered, since any other cut can be reduced to a cut of the same value by reducing its size to the nearest whole multiple of the lengths or widths of the pieces. Such a cut is called canonical, and a cut that contains only one type of small rectangle is called homogeneous. In addition, it can be shown that b) for each canonical inhomogeneous cut R, there is a canonical cut with the same cost, for which the coordinate of the first cut line is equal to at most half the corresponding size R. This allows to reduce the number of cuts considered by half.

At the beginning of the procedure, it is assumed that $l_i \leq L$ and $\omega_i \leq W$. However, when R is already cut, this relationship will not be valid for the resulting rectangles after the cut. Let l and ω be $(n \times 1)$ vectors with components l_i and ω_i.

There is a finite number of vectors z of dimension $(1 \cdot n)$ consisting of non-negative integers such that $zl < L$ and $z\omega < W$. Let P and Q be the corresponding sets of values of zl and $z\omega$. Then, appropriate sets are constructed for each $x \in P$ and $y \in Q$ to obtain feasible rectangles. An upper bound on the cost of cutting each rectangle can be determined for each cut. If the cost of a rectangle is proportional to its area, then the upper bound on the cost of a rectangle (α, β) is $U(\alpha, \beta) = \alpha \times \beta$. This upper bound on cost can be used to determine whether the rectangle should be cut further or not. Properties a) and b) and the value of the upper bound on the cost are used several times to obtain the optimal cut R (Kerimov and Abbasov, 2016).

Search tree method

A search tree procedure similar to the procedure described above was developed to solve the problem (B2), taking into account restrictions for each type of piece. Restrictions on the upper limit can easily be taken into account when solving one-dimensional problems of material cutting using the search tree method. However, when solving two-dimensional problems with upper -bound restrictions, these procedures are not very effective. Therefore, there is a need to develop improved procedures to solve this problem. Such a procedure is a search tree procedure. As it is shown in (Abbasov, 2015), this procedure is very effective in solving problems with an average number of pieces and consists of the following: the branches emanating from the root of the tree correspond to all possible cuts that can be made on the rectangular material. The node at the end of the branch corresponds to the rectangles resulting from the corresponding cut. In the subsequent knot, one of the rectangular pieces is selected to make the next cuts. Thus, a tree node corresponds to all rectangles resulting from cuts according to the paths from the tree root to this node. The cuts can be made along the length (x – cut) or width (y – cut) of the rectangular piece. The x – cut (or y – cut) is made so that the sum of the lengths (respectively, widths) of one or more pieces can be precisely adjusted to the resulting length or width of the rectangular material. Such cuts are called normal. The procedure for constructing normal cuts is based on properties a) and b). In addition, it should be considered that among such cuts, no two should produce the same patterns. This requirement is easy to implement if to make cuts of non-decreasing size. Cutting can be also stopped at a node if it is undesirable or impossible to cut any rectangle at that node. If at some node all the rectangles are not cut further, then no branches come out of this node. An elegant feature of this procedure is the way it determines the cost limits. An upper bound on the cost of the solution is obtained for each node. This is done as follows. At each node, there are rectangles that are not cut further, and rectangles that are candidates for further cutting (branching). To determine the best permissible arrangement of pieces in rectangles that do not branch (are not cut), a standard program for solving the transport problem is used (Ragimova, 2013). For each of the rectangles that will still branch, to determine the best arrangement of pieces in these rectangles, a DC procedure of the type described in solving Problem B2 is used. This arrangement is obtained by relaxing the constraint on the maximum desired number of pieces of each type, as a result of which the arrangement of pieces obtained by the search tree method may turn out to be unacceptable. If it turns out to be valid, then the resulting location is the best possible, and no further branching is needed from this node. Otherwise, this node is a candidate for further branching. Therefore, the current best feasible solution must be compared with the solution after each branching. Nodes where the upper cost limit is less than the cost of the current solution should also be cut off. Various rules can be used to branch from nodes. The procedure ends when there are no more nodes left to branch.

Heuristic method

Heuristic rules that can be used to create valid templates (placements) are as follows: a) start packing with the largest size, then the next smaller one, etc.; b) start packaging with the smallest size, then the next larger one, etc.; c) start from the corner (usually the bottom left); d) start the package from the edge and move inwards or e) start from the center and move towards the edges. A heuristic procedure based on rule c) was proposed in the work of Kerimov (1999) to solve the problem (B1) related to determining locations for loading a transport pallet. When loading, only one one-piece size is taken into account. The heuristic is to position the load along the four edges of the pallet. These placements are determined by the DC method. The piece can be placed from the edge along the length or width. The placements obtained along the edges are built up inward so that there is no overlap in the center. If overlap occurs in the center, the depth of the internal placements changes.

Dialogue graphic method

Let' us consider another new solution procedure (B2), based on an interactive graphical solution method using a digital computer. As it is already mentioned, the main difficulty in solving (B2) is that the templates must be valid. However, if the constraint in the problem (B2) is replaced by the constraint.

$$a_1q_1 + a_2q_2 + ...+ a_mq_m \leq A - \in \quad \text{(B3)}$$

It is easy to see that admissible templates can be obtained from solution (B3) for some $\in \geq 0$. For rectangular or triangular pieces one can assume $\in = 0$, but for pieces of other shapes one can assume $\in > 0$, which allows many unacceptable templates to be excluded from consideration. By varying $\in$, one can obtain different solutions to (B3). These solutions can then be checked on a digital computer plotter and a valid solution can be selected from them. Thus, the feasible solution (B2) is obtained by the dialog method using a graphical terminal. The advantage of this method is that it can be used for templates of any shape. Knowing the upper and lower bounds of $\in$ for a given template cut shape can be very helpful in this procedure. The bounds $\in$ available for some template shapes are given and the application of this procedure is described in the work of Ragimova (2013).

Methods for solving problems of cutting materials

Linear programming

If we remove the integer requirement in (A1), then the resulting problem is a linear programming (LP) problem, which is called problem (A2). Note that if x_j are not necessarily integers, then solution (A2) is relatively easier than (A1). Moreover, in many cases, especially for large values of N, the values of x_j are large enough that rounding to integer values (that satisfy the constraints) has little effect on the value of the objective function. Therefore, solution (A2) turns out to be almost equivalent to solution (A1).

Problem (A2) can be solved by the column construction method described in (Dyson and Gregory, 1974). This approach was proposed for guillotine cutting. Let column $(q_1,\ldots, q_m)$ be a valid template. If $v_1,\ldots, v_m$ are simplex factors corresponding to the optimal solution (A2), then the column $(q_1,\ldots, q_m)$ is determined from the solution to the problem (B2) of creating a template. To solve (B2), a two-stage procedure is used. Consequently, to solve problem (A2), the LP procedure can be used, in which problem (B2) must be solved at each iteration.

In another formulation a two-stage guillotine cutting problem is proposed, which is solved as a two-stage LP problem. The first stage corresponds to the process of cutting the rectangular material into long strips, the width of which corresponds to the width of the required rectangles, and the second stage corresponds to the process of cutting the strips into rectangles of the required length. The resulting LP problem can be solved directly using an algorithm for solving a one-dimensional cutting problem or using the decomposition procedure (Rahimova and Mansurova, 2022).

Heuristic procedures

Heuristic procedures are also used to solve two-dimensional problems. For problem (A1), the general heuristic procedure is as follows:

Step 1. Select template acceptance criteria. One such criterion that a uniform template must satisfy is based on the proportions of the maximum cut width and the segment scraps. According to this criterion, a valid template must satisfy the specified proportions. This admissibility criterion is illustrated in Figure 2. Appropriate acceptance criteria can be selected for other types of cuts.

Step 2. Combinations of pieces of only one type are tried, and templates that satisfy the selected criterion are chosen, if they exist.

Step 3. Combinations of two types of pieces are tried. Templates that satisfy the selected criterion are chosen.

Step 4. Combinations of pieces of three types are tried and templates are chosen that satisfy the selected criterion. If desired, combinations of more types can be tried. An example of a homogeneous template obtained using this procedure, consisting of three types of pieces, is shown in Figure 2.

Step 5. If the best templates obtained cover all the required pieces, the procedure stops. Otherwise, the criterion weakens and proceeds to step 2. The procedure continues until a sufficient number of templates is obtained to cover all the pieces.

This heuristic procedure was used for problems with homogeneous templates. In the work of Adamowicz and Albano (1976) it is assumed that the required number of pieces of each type is limited, and a modification of the above heuristic procedure is used. The procedure differs from that described here in that if the required placement cannot be found for a sheet of a given size, then an incomplete sheet is taken and combinations of pieces that are placed on this incomplete sheet are tried. In the incomplete stage, only guillotine placements are taken. Placements of this type are shown in Figure 3. After trying all combinations, the best combination for the partial sheet is selected using the DP-based procedure.

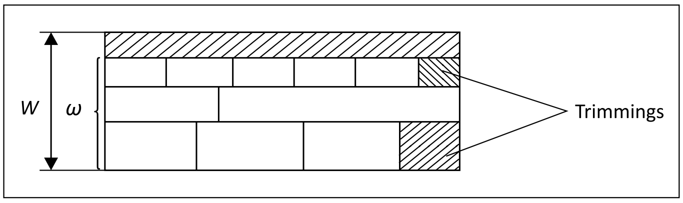

Figure. 2. Admissibility criterion (ω/W) – width criterion

Rysunek 2. Kryterium dopuszczalności (ω/W) – kryterium szerokości

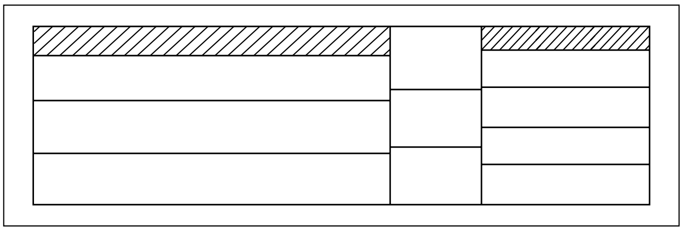

Figure 3. Typical placement obtained heuristically

Rysunek 3. Typowe rozmieszczenie uzyskane heurystycznie

Dialog optimization

The dialog procedure for solving two-dimensional problems of cutting material is a combination of the LP procedure and the dialog graphic procedure for creating a template (column of matrix Q). An application of this procedure to the problem of placing circular disks on a circular plate is given in (Aliyev, 2023). This interactive optimization procedure is also used for cutting rectangular pieces from rectangular material. The resulting placements are shown in Figure 4. This procedure has several advantages. Firstly, it can be used for any template shape. Secondly, it is very similar to the practical way of creating templates. In addition, this procedure allows the designer to have the best possible template ready to use, rather than having to recognize templates in a heuristic way. Any type of template can also be created using this procedure, including free-cut templates.

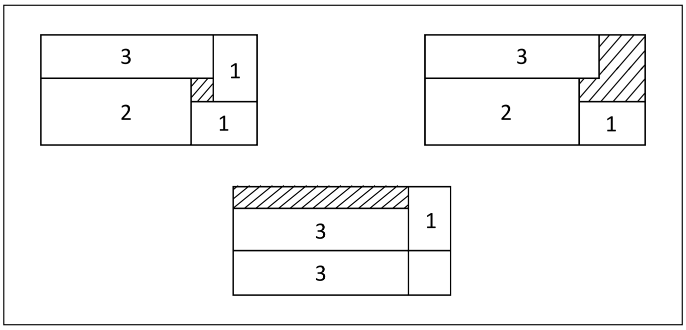

Figure 4. Optimal templates obtained using the dialog procedure
Rysunek 4. Optymalne szablony uzyskane za pomocą procedury dialogowej

Table 1. Example: size of large rectangle = (8.5)
Tabela 1. Przykład: rozmiar dużego prostokąta = (8,5)

Size of small rectangles	Need
(3.2)	70
(5.3)	40
(6.2)	60

Modification and applications

Until now, the basic methods for solving problems of cutting materials have been considered. However, in practice, several related issues arise that require modification of the basic techniques for special applications. These modifications and applications are discussed below.

Some practical issues in the two-dimensional material cutting problem are as follows:
a) Leaf orientation or structure.
b) There is material that may not be of the same quality everywhere. Some parts may have defects and some parts may be of better quality than others. In this case, the goal is to minimize the cost of the resulting pieces.
c) The shape of the cut pieces is not necessarily rectangular. Cutting into irregularly shaped pieces – a fairly common situation when cutting fabric, leather, and sheet metal.
d) Available material may be of different sizes.
e) Once the optimal templates for cutting small rectangles from several materials have been determined, the order in which these templates are used becomes important from a practical point of view. Indeed, the order will matter if different machine settings are required for the production of different templates and the cost of adjustments is very high. Therefore, setup costs may influence the determination of template types. In such a situation, the goal of optimization is to minimize the number of trims and the cost of adjustments.

The orientation of the sheet must be taken into account in order to determine where to make the cut. However, when the orientation of the sheet is already determined, then to define which templates to cut from the sheet, the appropriate technique is used. A relatively more difficult case occurs when individual areas on the sheet are defective (and can be easily directed before cutting). It is assumed that these defective places are limited by rectangles and are not allowed to enter the embedded material. The purpose of optimization is to obtain the largest number of pieces with minimum waste. In this case, the DC procedure can be used to determine the best templates. However, it is difficult to apply a recursive equation of a DC problem (B2) to solve this problem. However, you can try to apply the one-dimensional version of the recursive equation.

With a one-dimensional formulation, the sheet is divided along the length into parts which in turn are cut into strips. The cost of each part is determined by placing only one type of template. The presence of defects is taken into account when assessing this part, defects affect the number of pieces that give the maximum cost. The detailed procedure for solving this problem is given in (Nemhauser, 1967). In addition, in this task, it is proposed that individual sections should be made short to speed up the procedure for use in a real production process.

As for the problem of obtaining the maximum possible number of irregularly shaped pieces from a given number of pieces of material: since pieces of irregular shape are allowed, the cuts can be of the most general type (arbitrary). This problem is considered in (Nemhauser, 1967) in connection with the problem of template marking. To simplify, the problem can be solved by approximating irregular shapes with several rectangles as follows. Various irregular figures are combined into groups of two or three figures. These groups are selected in such a way that the cutting area is minimal. Grouped shapes are placed in rectangular area shapes. The placement of these rectangular shapes is then adjusted so that they fit within the boundaries of the sheet. The original problem is reduced to the task of cutting rectangular pieces from a given amount of material. The goal of optimization is to maximize the cost of the pieces.

As for the function linearly dependent on the area, the formulation of the problem is similar to the problem (B1). The difference in this case is that the cuts can be of any type. The developed procedure allows the creation of optimal templates for materials of multiple sizes in one pass of the procedure without restrictions on the number of pieces of each type obtained. If the cost of the various shapes is known in advance, then the rectangles can be ordered so that $v_1/a_1 > ... > v_m/a_m$. Otherwise, any appropriate order can be chosen.

For this problem, the recursive DP equation has the form:

$$F_i(L_x, W_x) = \max \begin{cases} v_i + \max F_i\left(\sum remaining\ rectangles\right), \\ F_{i-1}(L_x, W_x) \end{cases}$$

DP recursion develops in such a way that only one of the forms is placed at each stage. Namely, at each stage, only one placeable rectangle, starting from $i = 1$, is placed on a hypothetical sheet (called index rectangle), which is specified by the index pair (L_x, W_x). This rectangle can be placed at the corner of the index rectangle or at any point inside. The latter case is more general and provides more combinations for placing the remaining rectangles. At each stage, the dimensions of the index rectangle increase from a unit square (1.1) to the maximum dimensions (L, W) in unit increments. At stage i, one of the following options may occur for each set of values (L_x, W_x).

1. The dimensions (l_i, w_i) of the allocated rectangle are larger than the index rectangle.
2. The index rectangle and the placement rectangle are equal in size.
3. There is only one rectangle left in the x or y direction.
4. More than one rectangle remains in the x and y direction.

These options, together with the corresponding values of $F_i(L_x, W_x)$ are shown in Figure 5. Here $F_{(i-1)}(L_x, W_x)$ denotes that there is no allocation for the current values of (L_x, W_x) and $F_i(\sum remaining\ rectangles)$ gives the values of F_i to all rectangles resulting from placing the rectangle (l_i, ω_i) on the index rectangle. The recursive DP procedure at each placement provides an optimal arrangement of rectangles in (L_x, W_x), i.e., this procedure results in an optimal arrangement in index rectangles with sizes from (1.1) to (L, W). If L and W are the maximum length and maximum width, respectively, among all dimensions, then this procedure gives the optimal arrangement for all dimensions of the material.

Another procedure for solving a problem with free-form templates is the interactive graphical procedure described above.

Let us now consider the above modification in paragraph "d", i.e. the option where there is material of various sizes, and not just one, as in problem (A2). In this case, it is required to obtain a given number of rectangular pieces of various specified sizes from rectangular pieces of material of various sizes. An application of this problem is discussed in (Nemhauser, 1967). Considering the cuts to be guillotine, this problem can be solved by a simple generalization of the procedure for solving the problem (A2). The generalization consists of solving more knapsack problems at each iteration. If the number of

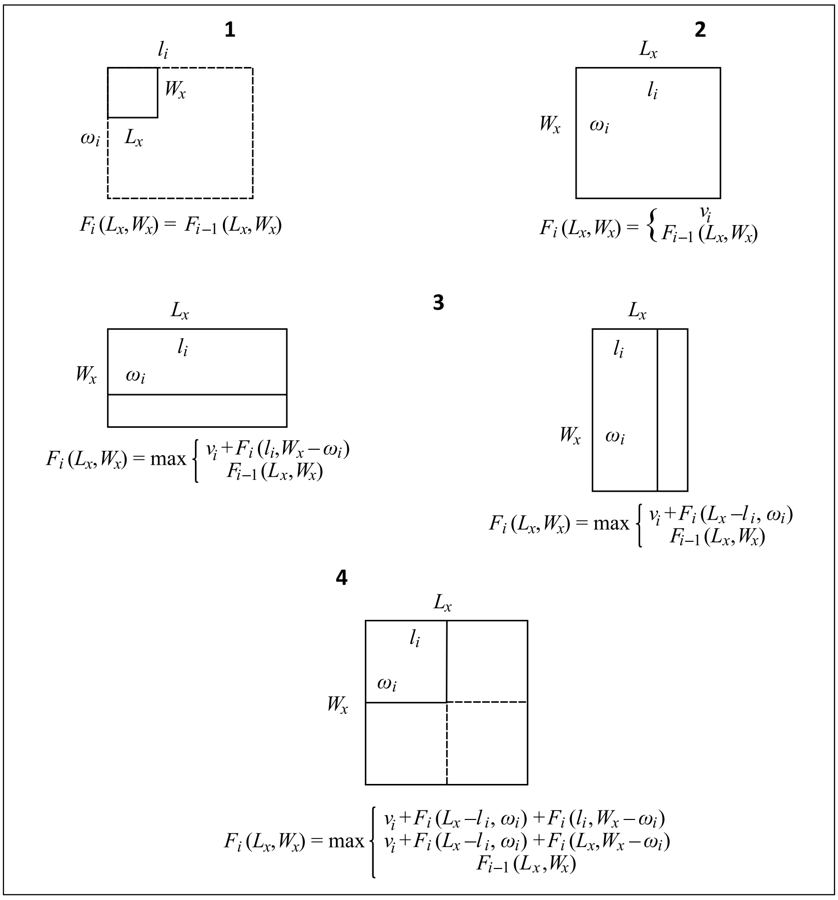

Figure 5. Possible results with recursion using the DP method

Rysunek 5. Możliwe wyniki z rekurencją wykorzystującą metodę programowania dynamicznego (DP)

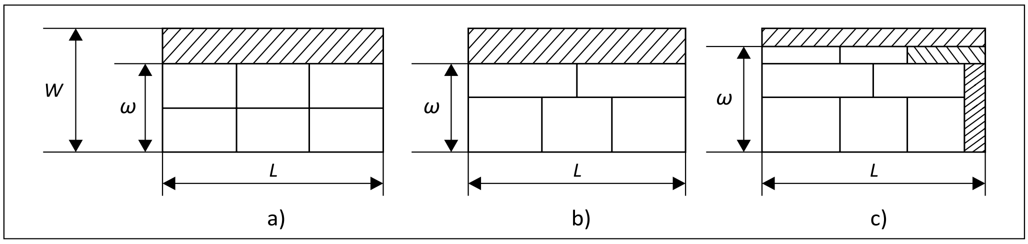

Figure 6. Construction of assembled workpieces; ω – workpiece width, l – workpiece length

Rysunek 6. Konstrukcja montowanych elementów; ω – szerokość elementu, l – długość elementu

different material sizes is large, this procedure requires a very long computation time. However, it turns out that if the same procedure is applied in a different way, its use becomes quite effective. This option is discussed below.

The idea here is to reverse the procedure. Namely, for a given need for rectangular pieces, a minimum number of workpieces of various sizes are first assembled, assuming that the assembled workpieces do not exceed the given maximum lengths and widths of the material. Then sheets of material are cut to dimensions equal to or close to the dimensions of the workpieces to obtain pieces of specified sizes. This procedure will result in minimal waste. The operation of assembling workpieces can be called primary, and the operation of cutting pieces of given sizes from sheets of material with dimensions equivalent to the dimensions of the assembled workpieces can be called secondary.

As it can be seen in Fiqure 6, for the special application described in (Aliyev and Aliyeva, 2017), the process of assembling blanks from small rectangular pieces involves uniform cuts. In Figure 6a, all but one type of piece was used, in Figure 6b – two types, and in Figure 6c – three types (the dimensions of the assembled workpiece are also shown here). In general, any number of types of pieces can be used to assemble a workpiece. From the point of view of cutting material, this assembly process is identical to the construction of templates, but these templates must be uniform. Uniform templates are based on special types of guillotine cutting of material into strips, each of which corresponds to only one type of rectangular piece. Thus, the placement of the view shown in Figure 7 is unacceptable.

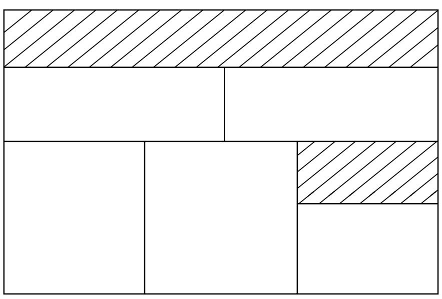

Figure 7. Inappropriate placement

Rysunek 7. Niewłaściwe ułożenie

The problem for homogeneous templates is relatively simpler than the problem for the general type of guillotine cutting. In general, for various applications, guillotine cutting is applicable in the primary operation of constructing assembled workpieces. For the application considered, the primary problem reduces to problem (A2), except that the resulting placements must be homogeneous, as it is shown in Figure 6. This problem can be solved by the LP procedure used to solve (A2).

Once the dimensions of the workpieces are determined, the secondary operation is reduced to selecting the source material that is closest in size to the assembled workpieces and cutting.

The templates obtained by solving the problem (A2) according to the minimum waste criterion will, in the general case, be statically random. The difficulty here is that when sorting through such templates during the cutting process, the transition from one template to another may require a sharp readjustment of the machine. Additional readjustment is also required if the material has different dimensions. This leads to a new interesting aspect of the problem of cutting material, namely minimizing setup time. Dyson and Gregory (1974) considered this problem in relation to cutting sheets of glass.

There are two approaches to solving this problem. The first is to determine the optimal templates using the LP procedure and then arrange them so as to obtain the minimum readjusting costs. The second approach is to use a procedure that simultaneously minimizes waste and readjusting time.

In the first approach, to minimize the readjusting time in the production of templates obtained by the LP procedure for the problem (A2), a procedure like the one for solving the traveling salesman problem can be used. In this case, each template is considered as a city, and the cost matrix when ordering templates can be determined by the number of new pieces that must be cut for successive templates. The number of new pieces that must be cut if template A is followed by template B is equal to the number of pieces contained in template A (not B). A fictitious template is used to properly count new pieces. However, this procedure can be very time-consuming when it comes to computations.

Some heuristic procedures for implementing the second approach to determine templates that minimize waste and readjusting costs are discussed.

Conclusion

Two-dimensional problems of material cutting are quite common in practice. This paper examines several techniques available in the literature for solving many of these problems. With the increasing applications of digital computers to control production processes, these techniques can be easily programmed and used to make more accurate and cost-effective decisions.

References

Abbasov E.I., 2015. To the question of increasing the performance of sucker rod pumps. Collection of abstracts of the 69th international youth scientific conference. *Oil and Gas, Moscow*, 2: 46–51.

Adamowicz M., Albano A., 1976. A Solution of the Rectangular Cutting-Stock Problem. *IEEE Transactions on Systems, Man, and Cybernetics*, MSC-6(4): 302–310. DOI: 10.1109/TSMC.1976.5408780.

Aliyev A.M., 2023. Determination of kinematic parameters of the compressor unit applied to low-pressure petroleum gas. *Nafta-Gaz*, 79(4): 252–256. DOI: 10.18668/NG.2023.04.03.

Aliyev A.M., Aliyeva S.Y., 2017. Constructions of oilfield equipment and special facilities. *Azerbaijan State Oil and Industry University, Baku*.

Aliyeva S.Y., Demirova C.R., Abbasova S.A., 2021. Research of the problem of optimization and development of a calculation method for two-stage chain drives used in heavy industrial vehicles in conditions of economic efficiency. *EUREKA: Physics and Engineering*, 4: 105–112.

Dyson R.G., Gregory A.S., 1974. The Cutting Stock Problems in the Flat Glass Industry. *Operational Research Quarterly*, 25(1): 41–53.

Golden B.L., 1976. Approach to Cutting Stock Problem. *AIIE Trans.*, 256–272.

Karimov O.M., Abbasov E.I., 2016. Study of the influence of the plunger position inside the cylinder on the operation of the pump. *News of the Azerbaijan Engineering Academy*, 2: 56–61.

Kerimov O.M., 1999. Increasing the performance and operational parameters of sucker rod pumps. *Maarif, Baku*.

Kerimov O.M., Abbasov E.I., 2016. Determining the friction force between the guide and the draw rod in an insert-type downhole pump. *Equipment and technologies for the oil and gas complex. IRIOEOG*, 2: 82–87.

Nemhauser G.L., 1997. Introduction to Dynamic Programming. *Wiley, New York*.

Ragimova M.S., 2013. Calculation of the thickness of seals for flange joints of oilfield equipment. *Oilfield Business*, 5: 36–38.

Rahimova M.S., Mansurova S.I., 2022. Methods for calculating the fatigue and strengths machine parts. *Nafta-Gaz*, 78(12): 888–892. DOI: 10.18668/NG.2022.12.06.

Vahidov M.A., Karimov O.M., Eyvazova Z.E., 2008. Oil and gas production technique. *Azernashr, Baku*, 458.

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