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The influence of rock heterogeneity on the stability of the well wall

Wpływ niejednorodności skał na stabilność ściany odwiertu

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ABSTRACT: One of the complications that arise while drilling wells is hydraulic fracturing. Hydraulic fracturing pressure is the pressure at which the integrity of the rock in the walls of the well is compromised, leading to the formation of artificial cracks. Various authors have proposed formulas to determine the hydraulic fracturing pressure P_{hf} in the absence of actual data.

$$P_{hf} = 0.87\ P_{rock}$$
$$P_{hf} = 0.85\ (P_{rock} - P_{res}) + P_{res}$$
$$P_{hf} = [\mu/(1 - \mu)]\ (P_{rock} - P_{res}) + P_{res}$$
$$P_{hf} = [2\mu/(1 - \mu)]\ P_{rock}$$

where: μ – Poisson's ratio, which takes values of 0.25–0.4 for dense clay, 0.33–0.4 for clay with sandstone interlayers, 0.1–0.2 for shales, 0.3–0.35 for sandstone, 0.28–0.33 for limestone; P_{rock}, P_{res} – rock and reservoir pressure, respectively. This study examines the problem of determining hydraulic fracturing pressure taking into account the heterogeneity of rocks. The work can be roughly divided into two parts. In the first part, the physical properties of porous bodies in contact with fluids are studied, treating them as a two-phase medium. When a porous body comes into contact with fluids, the fluids penetrate into them, transforming them into a two-phase medium. The term "fluids" refers to both liquids and gases, with real liquids considered incompressible and gases highly compressible. The remaining part of the porous body, excluding the pores, is called the skeleton. Pores in porous bodies are connected through capillary tubes. In the second part, the influence of the magnetic field and rock heterogeneity on the stability of the well wall is studied. Formulas are derived to determine hydraulic fracturing pressure, depending on the mechanical properties of the rocks and the reservoir pressure. Based on the theory of destruction, the critical value of excess pressure (or hydraulic fracturing pressure) is determined, which is necessary to ensure the safety of drilling oil and gas wells. The machines used in drilling operations in the oil and gas industry must be easy to operate, reliable, and capable of long-time use. When designing such machines, considerations such as being lightweight, economical, and quick and inexpensive to prepare should be taken into account from the outset.

Key words: heterogeneity, reduced value, pores, physical properties, well walls, failure condition, critical value, hydrostatic pressure.

STRESZCZENIE: Jednym z problemów występujących podczas wykonywania odwiertów jest szczelinowanie hydrauliczne. Ciśnienie szczelinowania hydraulicznego to ciśnienie, przy którym dochodzi do naruszenia integralności skał w ścianach odwiertu, co prowadzi do powstawania sztucznych szczelin. Istnieje kilka wzorów opracowanych przez różnych autorów, pozwalających określić ciśnienie szczelinowania hydraulicznego P_{hf} w przypadku braku rzeczywistych danych.

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gdzie: μ – współczynnik Poissona, który przyjmuje wartości od 0,25 do 0,4 dla iłowców, od 0,33 do 0,4 dla iłowców z przewarstwieniami piaskowca, od 0,1 do 0,2 dla łupków, od 0,3 do 0,35 dla piaskowców, od 0,28 do 0,33 dla wapieni; P_{rock}, P_{res} – odpowiednio ciśnienie w skale i ciśnienie złoża. Niniejsze badania dotyczą problemu określenia ciśnienia szczelinowania hydraulicznego z uwzględnieniem heterogeniczności skał. Praca ta dzieli się zasadniczo na dwie części. W ramach pierwszej części przebadane zostały właściwości fizyczne ciał porowatych w kontakcie z cieczami, traktując je jako ośrodek dwufazowy. Gdy ciało porowate styka się z cieczami, te przenikają one do niego, przekształcając je w ośrodek dwufazowy. Termin „ciecze" odnosi się zarówno do cieczy, jak i gazów, przy czym rzeczywiste ciecze są uznawane za nieściśliwe, a gazy za wysoce ściśliwe. Pozostała część ciała porowatego, z wyjątkiem porów, nazywana jest szkieletem. Pory w ośrodkach porowatych są połączone przez kapilary. W ramach drugiej części analizowany był wpływ pola magnetycznego oraz heterogeniczności skał na stabilność ścian odwiertu. Opracowano wzory na określenie ciśnienia szczelinowania hydraulicznego, zależnie od właściwości mechanicznych skał oraz ciśnienia złożowego. Na podstawie teorii zniszczeń określono krytyczną wartość nadciśnienia (lub ciśnienia szczelinowania hydraulicznego), co jest niezbędne do zapewnienia bezpieczeństwa podczas wykonywania odwiertów naftowych i gazowych. Maszyny stosowane w operacjach wiertniczych w przemyśle naftowym i gazowym muszą być

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łatwe w obsłudze, niezawodne i przystosowane do długotrwałej eksploatacji. Projektując takie maszyny należy od samego początku brać pod uwagę takie aspekty jak niska waga, ekonomiczność, a także możliwość ich szybkiego i niskonakładowego przygotowania do pracy.

Słowa kluczowe: heterogeniczność, wartość zredukowana, pory, właściwości fizyczne, ściany odwiertu, stan zniszczenia, wartość krytyczna, ciśnienie hydrostatyczne.

Introduction

We now derive formulas for the introduced physical properties of two-phase media. Let us accept the following notation. Porous body volume – V; skeletal volume – V_s; volume of fluid in the pores – V_l; skeleton mass – m_s; the mass of the polymer whose pores are filled with fluid – m; mass of fluid in pores – m_l; density of the skeleton material – ρ_s; density of a polymer whose pores are filled with fluid – ρ; density of pore fluid – ρ_f. Three of these nine quantities – V, m, and m_s – can be measured directly. Two others, ρ_f and ρ_s, are known. The remaining quantities can be determined through these three quantities as follows (Horsrud 2001; Al-Ajmi and Zimmerman, 2005; Habibov et al., 2022).

$$m_l = m - m_s \quad (1)$$

$$m_l = V_l/\rho_l = (V - V_s)/\rho_l \quad (2)$$

$$V_s = V - V_l \quad (3)$$

$$\rho_s = m_s/V_s = m_s/(V - V_l) \quad (4)$$

From equation (1)

$$m_l + m_s = m. \text{Thus, we have } (m_l/m) + (m_s/m) = 1 \quad (5)$$

Multiply both sides of equation (5) by the cross-sectional area S, we get:

$$m_l/m_S + m_s/m_S = S$$

The first term on the left side of this equation represents the cross-sectional area of the fluid column, and the second term is the cross-sectional area of the skeleton.

Next, we derive an expression for the reduced Young's modulus of a polymer whose pores are filled with fluid. As is known, under uniaxial tension, Hooke's law takes the form:

$$\varepsilon = \sigma/E;\ \sigma = F/S\ ;\ \varepsilon = F/SE \quad (6)$$

where:
E – Young's modulus,
σ – tensile stress,
F – tensile force,
ε – elongation.

When the rod is compressed, the compression deformations of the liquid column in the skeleton and pores are equal to each other, that is, $\varepsilon_l = \varepsilon_s$. Therefore:

$$T_s/(S_s \cdot E_s) = T_l/(S_l \cdot E_l) \quad (7)$$

where: T_s and T_l – compressive forces acting on the cross--section of the skeleton and pores, respectively, S_s, E_s and S_l, E_l – cross-sectional areas of the skeleton and fluid, respectively, and Young's modulus. On the other hand, the compression of both the skeleton, the fluid column, and the general element are equal to each other, i.e.

$$\frac{T_s}{S_s E_s} = \frac{T_l}{S_l \cdot E_l} = \frac{T_s + T_l}{(S_s + S_l) E_r} \quad (8)$$

where:
E_r – reduced Young's modulus.

From the last equation:

$$E_r = \frac{m_s E_s + m_l E_l}{m} \quad (9)$$

Thus, we obtain the formula for defining the Young's modulus for a two-phase medium.

The following expressions can be similarly derived for the reduced sliding modulus G_r, Poisson's ratio ν_r, tensile strength σ_{tr}, creep kernel K_r, and relaxation kernel Γ_r.

$$G_r = \frac{m_s G_s + m_l G_l}{m} \quad (10)$$

$$\nu_r = \frac{m_s \nu_s + m_l \nu_l}{m} \quad (11)$$

$$\sigma_r = \frac{m_s \sigma_{sB} + m_l \cdot \sigma_{tl}}{m} \quad (12)$$

$$K_r = \frac{m_s K_s + m_l \cdot K_l}{m} \quad (13)$$

$$\Gamma_r = \frac{m_s \Gamma_s + m_l \Gamma_l}{m} \quad (14)$$

In equations (10)–(14), quantities with index s refer to the skeleton, and quantities with index l – to fluid. Since liquids and gases take the shape of the container into which they are poured, the sliding modulus of liquids and gases can be considered zero. Thus, equation (10) takes the following form.

$$G_r = \frac{m_s G_s}{m} \quad (15)$$

Considering that $m = m_s + m_l$ and $G_s = \frac{E_s}{2(1+\nu_s)}$ equation (15) becomes:

$$G_r = \frac{m_s}{m_s + m_l} \cdot \frac{E_s}{2(1+\nu_s)} \quad (16)$$

As can be seen from this formula, with increasing fluid mass in the pores, the shear modulus decreases. However, the shear modulus depends on the mechanical properties of the skeleton, not on those of the liquid or gas filling the pores. Considering that $m = m_s + m_l$, equation (9) gives us:

$$E_r = \frac{m_s E_s + m_l E_l}{(m_s + m_l)^2} \tag{17}$$

Since the volume of the skeleton during deformation is small, and the pore volume changes significantly, if we treat m_l as a variable in equation (17) and take the derivative of E_r with respect to m_l:

$$E_r' = \frac{m_s (E_l - E_s)}{(m_s + m_l)^2} \tag{18}$$

As can be seen from (18), E_r' is positive when $E_l > E_s$, and E_r' is negative when $E_l < E_s$. This means that regardless of the fluid mass in the pores, when the Young's modulus of the fluid is greater than the Young's modulus of the skeleton, the reduced Young's modulus increases with increasing mass of the fluid and, conversely, decreases with increasing mass of the fluid.

If the fluid in the pores of the polymer is incompressible, $\nu_L = 0.5$. In this case, from equation (11):

$$\nu_r = \frac{m_s \nu_s + m_l \cdot 0.5}{m_s + m_l} \tag{19}$$

From the physical properties of the material, the coefficient of thermal expansion α and specific heat capacity c are determined as follows:

$$\alpha_r = \frac{m_s \propto_s + m_l \propto_l}{m_s + m_l} \tag{20}$$

$$c_r = \frac{m_s c_s + m_l c_l}{m_s + m_l} \tag{21}$$

Statement and solution of the problem

Let us consider the stressed state of the well, taking into account the heterogeneity of rocks, and determine the hydraulic fracturing pressure.

Expression (17), defining the Young's modulus of a two-phase medium, can be shown in the following form:

$$E_r = \frac{E_s v_s + E_l v_l}{v_s + v_l} \tag{22}$$

where:
E_r – reduced Young's modulus of a two-phase medium,
E_s – Young's modulus of the solid skeleton phase,
E_l – Young's modulus of the liquid phase,
v_s – volume of the solid phase,
v_l – pore volume.

It is known that the porosity coefficient e_0 of the rock is determined by the following expression:

$$e_0 = v_j / v \tag{23}$$

where: $v = v_s + v_l$

Then, (22) will take the form:

$$E_r = E_s + (E_l - E_s)\, e_0 \tag{24}$$

Similarly, for the reduced Poisson's ratio ν_r, we obtain the expression:

$$\nu_r = \nu_s + (\nu_l - \nu_s)\, e_0 \tag{25}$$

where:
ν_s, ν_l – Poisson ratios of the solid and liquid phases, respectively.

It is known that the drilling process is carried out under pressure created by a column of drilling fluid, which is called hydrostatic P_{hs}, and is determined as follows:

$$P_{hs} = \rho g H \tag{26}$$

where: H is the drilling depth. To prevent formation fluid from entering the well, the hydrostatic pressure must be greater than the reservoir pressure P_{rp}. The required drilling fluid density at a known reservoir pressure is determined by the formula.

$$\rho = \frac{P_{rp} + \Delta P}{gH} \tag{27}$$

where:
ΔP is the excess pressure that is necessary for the pressure to exceed the reservoir pressure.

It is normatively established that this excess should be 10% of the formation value, but not more than 1.5 MPa.

The equilibrium equation for the near-wellbore zone has the form (Fjaer et al., 2008; Gere and Goodno, 2011; Kheyrabadi and Orujov, 2024):

$$\frac{d\sigma_R}{dr} + \frac{1}{r}(\sigma_R - \sigma_\varphi) = 0 \tag{28}$$

where:
σ_R, σ_φ – are the radial and tangential components of the stress vector, respectively. In the case under consideration, the generalized Hooke's law has the form:

$$\left.\begin{aligned} \sigma_R &= \frac{E_r}{a^2 - b^2}(a\varepsilon_r + b\varepsilon_\varphi) \\ \sigma_\varphi &= \frac{E_r}{a^2 - b^2}(b\varepsilon_r + a\varepsilon_\varphi) \end{aligned}\right\} \tag{29}$$

where:
ε_R and ε_φ –radial and tangential components of the relative deformation, respectively, $a = 1 - \nu_r^2$, $b = \nu_r + \nu_r^2$.

It is known that the expressions defining the dependence of relative deformations on displacements have the form:

$$\left.\begin{aligned}\varepsilon_R &= \frac{du}{dr}\\ \varepsilon_\varphi &= \frac{u}{r}\end{aligned}\right\} \quad (30)$$

If we take into account (30) in (29), we obtain:

$$\left.\begin{aligned}\sigma_R &= \frac{E_r}{a^2-b^2}\left(a\frac{du}{dr}+b\frac{u}{r}\right)\\ \sigma_\varphi &= \frac{E_r}{a^2-b^2}\left(b\frac{du}{dr}+a\frac{u}{r}\right)\end{aligned}\right\} \quad (31)$$

If we take into account (31) in (28), we obtain the following second-order differential equation for the displacement u:

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=0 \quad (32)$$

The general solution to equation (32) has the form:

$$u = C_1 r^{-1} + C_2 r \quad (33)$$

If we take into account (33) in (31), it turns out:

$$\left.\begin{aligned}\sigma_R &= \frac{E_r}{a^2-b^2}\left[(a+b)C_2-(a-b)\frac{C_1}{r^2}\right]\\ \sigma_\varphi &= \frac{E_r}{a^2-b^2}\left[(a+b)C_2+(a-b)\frac{C_1}{r^2}\right]\end{aligned}\right\} \quad (34)$$

where:

C_1, C_2 – integration constants, which are determined from the following boundary conditions.

$$\left.\begin{aligned}&\text{at } r=r_0 \quad \sigma_R=-P_{hs}-\Delta P\\ &\text{at } r=r_1 \quad \sigma_R=-P_{rp}\end{aligned}\right\} \quad (35)$$

$$\left.\begin{aligned}C_1 &= \frac{(a+b)}{E_r\left(r_0^2+r_1^2\right)}(P_{hs}+\Delta P-P_{rp})r_0^2r_1^2\\ C_2 &= -\frac{(a-b)}{E_r\left(r_0^2+r_1^2\right)}\left(r_0^2P_{hs}+r_1^2P_{rp}\right)\end{aligned}\right\} \quad (36)$$

For movement, the following expression is obtained:

$$u=\frac{(a+b)}{E_r\left(r_0^2+r_1^2\right)}(P_{hs}+\Delta P-P_{rp})r_0^2r_1^2\frac{1}{r}-\frac{(a-b)}{E_r\left(r_0^2+r_1^2\right)}\left(r_0^2P_{hs}+r_1^2P_{rp}\right)r \quad (37)$$

Thus, the stress-strain state will be determined as follows:

It is known that the condition for the destruction of a body under the influence of external forces has the form (Zhang et al., 2010; Salem and Nooh, 2014; Aslannezhad et al., 2015; Salem, 2016):

$$\sigma_R^2+\sigma_\varphi^2=2\sigma_{rm}^2 \quad (40)$$

where:

σ_{rm} – reduced tensile strength, which is defined as follows.

$$\sigma_{rm}=\sigma_{sm}+(\sigma_{lm}-\sigma_{sm})e_0 \quad (41)$$

where:

σ_{sm}, σ_{lm} – strength limits of the components of a two-phase medium.

Destruction of the well wall occurs at $r = r_0$. Therefore, we determine the stress at $r = r_0$:

$$\left.\begin{aligned}\sigma_R &= -P_{hs}-\Delta P\\ \sigma_\varphi &= \alpha(P_{hs}+\Delta P)-2\beta P_{rp}\end{aligned}\right\} \quad (42)$$

Considering that the formation radius r_1 is much larger compared to the well radius r_0, the values of α and β can be taken as follows:

$$\alpha=\frac{r_1^2}{r_1^2-r_0^2}\approx 1,\quad \beta=\frac{r_1^2+r_0^2}{r_1^2-r_0^2}\approx 1$$

Taking (42) into account in (40), we obtain a quadratic equation with respect to:

$$P_{hs}+\Delta P(P_{hs}+\Delta P)^2-2(P_{hs}+\Delta P)P_{rp}+2P_{rp}^2=\sigma_{rB}^2 \quad (43)$$

The solution to equation (43) has the form:

$$P_{hs}+\Delta P=P_{rp}\pm\sqrt{\sigma_{rB}^2-P_{rp}^2} \quad (44)$$

For the existence of real solutions to equation (43), the following condition must be met:

$$\sigma_{rB}^2-P_{rp}^2\geq 0$$

For any rock of the productive formation, this condition is satisfied:

$$P_{rp}\leq\sigma_{rB} \quad (45)$$

Considering (27):

$$\Delta P=P_{rp}+\sqrt{\sigma_{rB}^2-P_{rp}^2}-\rho gH \quad (46)$$

Chus, a formula was obtained that determines the critical value of excess pressure at a given reservoir pressure. This is necessary for safe drilling of oil and gas wells.

$$\left.\begin{aligned}\varepsilon_R &= \frac{du}{dr}=\frac{1}{E_r\left(r_0^2+r_1^2\right)}\left[-(P_{hs}+\Delta P-P_{rp})(a+b)\frac{r_0^2r_1^2}{r^2}-\left(r_0^2(P_{hs}+\Delta P)+r_1^2P_{rp}\right)(a-b)\right]\\ \varepsilon_\varphi &= \frac{u}{r}=\frac{1}{E_r\left(r_0^2+r_1^2\right)}\left[(P_{hs}+\Delta P-P_{rp})(a+b)\frac{r_0^2r_1^2}{r^2}-\left(r_0^2(P_{hs}+\Delta P)+r_1^2P_{rp}\right)(a-b)\right]\end{aligned}\right\} \quad (38)$$

$$\left.\begin{aligned}\sigma_R &= \frac{1}{\left(r_0^2+r_1^2\right)}\left[-\left(r_0^2(P_{hs}+\Delta P)+r_1^2P_{rp}\right)-(P_{hs}+\Delta P-P_{rp})\frac{r_0^2r_1^2}{r^2}\right]\\ \sigma_\varphi &= \frac{1}{\left(r_0^2+r_1^2\right)}\left[-\left(r_0^2(P_{hs}+\Delta P)+r_1^2P_{rp}\right)+(P_{hs}+\Delta P-P_{rp})\frac{r_0^2r_1^2}{r^2}\right]\end{aligned}\right\} \quad (39)$$

Conclusions

1. Expressions were obtained for the mechanical characteristics of polymers whose pores are filled with fluid.
2. With an increase in the fluid mass in the pores, the shear modulus decreases.
3. It has been theoretically proven that when the Young's modulus of the fluid in the pores is greater than the Young's modulus of the skeleton, the reduced Young's modulus increases as the mass of the liquid increases, and conversely, when the Young's modulus of the liquid is less than that of the skeleton, it decreases as the mass of the liquid increases.
4. When filling pores with an incompressible fluid, Poisson's ratio always increases with increasing mass of the fluid.
5. A formula was obtained that determines the critical value of excess pressure at a given reservoir pressure, taking into account the heterogeneity of rocks. This is necessary for safe drilling of oil and gas wells.

References

Al-Ajmi A.M., Zimmerman R.W., 2005. Relation between the Mogi and the Coulomb failure criteria. *International Journal of Rock Mechanics and Mining Sciences*, 42(3): 431–439. DOI: 10.1016/j.ijrmms.2004.11.004.

Aslannezhad M., Manshad A.K., Jalalifar H., 2015. Determination of a safe mud window and analysis of wellbore stability to minimize drilling challenges and non-productive time. *Journal of Petroleum Exploration and Production Technology*, 6(3). DOI: 10.1007/s13202-015-0198-2. 493-503.

Bonar A.S., 2016. Drilling Well Problems, 1–39 .

Fjaer E., Holt R., Horsrud P., Raaen A., Risnes R., 2008. Petroleum related rock mechanics. 2nd Edition. *Elsevier*, 53: 1–491.

Gere, J.M., Goodno, B.J., 2011. Mechanics of materials, Brief SI Edition. Cengage Learning, 1–644.

Habibov I.A., Abasova S.M., Sadigova T.Y., 2022. Assessment of the effect of ferromagnetic liquids on oil recovery of bituminous sands. *Eastern-European Journal of Enterprise Technologies*, 3(6(117): 42–52. DOI: 10.15587/1729-4061.2022.257104.

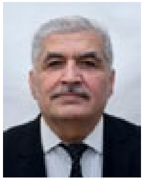
Horsrud P. 2001. Estimating mechanical properties of shale from empirical correlations. *SPE Drilling and Completion*, 16(2): 68–73. DOI: 10.2118/56017-PA.

Kheyrabadi Q., Orujov Y.A., 2024. The influence of rock heterogeneity on stability well walls. *International Journal of Humanities and Natural Sciences*, 7–2(94): 167–174. DOI: 10.24412/2500-1000-2024-7-2-167-174.

Salem A.M., 2016. Innovative Drilling Fluid Design Using Nano Materials. *8th Mediterranean Offshore Conference & Exhibition, Egypt*, 1–20.

Salem A.M., Nooh A., 2014. Reduction of Formation Damage and Fluid Loss using Nano-sized Silica Drilling Fluids. *Petroleum Technology Development Journal*, 2: 75–88.

Zhang L., Cao P., Radha K.C., 2010. Evaluation of rock strength criteria for wellbore stability analysis. *International Journal of Rock Mechanics and Mining Sciences*, 47(8): 1304–1316. DOI: 10.1016/j.ijrmms.2010.09.001.



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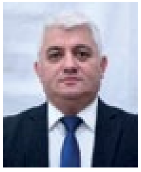

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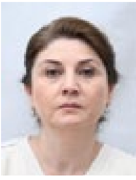

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